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mal " conditions. On the other hand, it is evident that this return to normal will not be violent, but gradual.

The interpretation of statistical data at any time is full of difficulties; at the present time the difficulties are extreme. If the fluctuations of statistical series return to normal, will the transition take a few months or several years? Is a tendency to return already evident? What are the peculiarities of current data? These are questions reserved for consideration in subsequent numbers of the REVIEW, and in the MONTHLY SUPPLEMENTS which will be devoted to the analysis of current statistics.

The Index of General Business Conditions presented has been constructed from the data for the period 1903 to 1914, and is offered as a guide to the relations that exist among economic phenomena in ordinary peace times. It obviously is of no value in interpreting the connection that existed during the Great War, but is offered in the belief that it will prove serviceable in the study of conditions following the establishment of peace. While the war may have changed many things, it probably has not altered the fundamental conditions which in ordinary times produce fluctuations in business activity. It may be expected, therefore, that, as conditions return to normal, the relations that existed between most, if not all, of our series of economic and business statistics will tend to be reestablished upon the basis which obtained from 1903 to 1914. To the extent that this expectation proves justified, the Index of General Business Conditions should prove a serviceable guide to the fundamental economic movements of the years that lie ahead.

The statements made in the preceding sections are conclusions. These conclusions have been presented, so far as possible, in non-technical language. They have been drawn, however, only by the application to business data of a considerable amount of mathematical and statistical technique. The soundness of the analysis and technique conditions the soundness of the conclusions reached. In accordance with scientific usage we shall give a complete explanation of the basis of our conclusions so that it will be possible for anyone interested to follow the argument step by step. This explanation, however, cannot be made entirely non-technical. The individual items used are themselves composite; they are symptomatic of the activity of a complex economic organization; the methods developed cover a large portion of the theory of statistics; the concepts involved demand careful examination; the nature of economic correlation and causation requires scrutiny.

Consideration of the nature of the data used, a detailed description of the statistical methods developed for correcting the original items and comparing the corrected series, and definitions of concepts, these are presented in Part II of this study. The attempt will be made to present the ideas and arguments clearly and non-technically. The complexities of the subject, how-

ever, make the difficulties of exposition considerable. We shall attempt to secure simplicity and accuracy in statement. When there appears to be a choice between the two, however, the latter will not be sacrificed to the former.

Anyone interested only in results and willing to accept our results on authority will find them summarized in the preceding sections, and in Section 3 of Part II; anyone desiring to examine in full the bases of our conclusions should read Part II, and study the data presented in Parts III and IV.

II. THE METHOD USED

I. RÉSUMÉ OF THE PRECEDING STUDY

In the January number of THE REVIEW OF ECONOMIC STATISTICS fifteen monthly series of business statistics were presented and analyzed. Each item was conceived to be a compound or composite, that is, each actual figure was conceived to be a magnitude determined by the concurrent action of distinct causes. A method was developed of measuring and eliminating those constituents of the items of time series ascribable to secular trend and seasonal variation. The method developed was applied to the fifteen series of business statistics covering periods of from sixteen to twenty-six years.

The need of eliminating secular trend and seasonal variation from statistical series before attempting to employ them as indices of business conditions, as well as the method which we have used in such elimination, may be made clear by a simple illustration. Our purpose is to judge the significance of items for different dates. Let us take the production of pig iron in the United States at any three dates some distance apart from each other. The tonnage of pig iron produced in the United States in March 1904 was 1,447,000, the tonnage in February 1908 was 1,077,000, while the tonnage in December 1918 was 3,434,000. Which one of these figures indicates the greatest relative activity of the iron industry, relative, that is, with respect to the facilities of the industry at the time in question? To answer this question we must take account of the fact that the country and the iron industry have grown in the interval from March 1904 to December 1918. It is obvious that if the growth were by equal monthly increments, a "normal" of 1,528,000 tons in March 1904, for instance, would not be normal in 1908 or 1918. On the assumption of uniform growth the normal of February 1908 would be 1,901,000 tons, and that of December 1918 would be 2,932,000 tons. The actual production figures for the three months are 1,447,000 tons, 1,077,000 tons, and 3,434,000 tons, respectively. The ratios of the actual figures to the corresponding "normal" figures are 95 per cent, 57 per cent, and 117 per cent, respectively. Omitting the percentage signs, the figures just given are termed indices. They are indices of pig-iron production so constructed, by the

TABLE B. — RANGE OF PERCENTAGE DEVIATIONS OF ORIGINAL ITEMS FROM SECULAR TREND
CORRECTED FOR SEASONAL VARIATION.

1. New York Clearings	—36 (December 1907) to +49 (January 1906)
2. Production of Pig Iron	—44 (December 1903, January 1908, May 1908) to +27 (December 1909)
3. Outside Clearings	—23 (December 1907) to +13 (January and May 1907)
4. Bradstreet's Prices *	—9.1 (June 1908) to +9.1 (March 1907)
5. Imports	—32 (March 1908) to +26 (December 1906)
6. Building Permits	—51 (March 1908) to +48 (August 1911)
7. Railroad Gross Earnings	—16 (July 1908) to +16 (May 1907)
8. Shares Traded *	—71 (June 1904) to +128 (January 1906)
11. Business Failures (Bradstreet)	—27 (January 1910) to +34 (January 1908)
12. Yield of 10 Railroad Bonds*	—4.1 (April and May 1909) to +11.3 (November 1907)
13. Rate on 4-6 Months Paper	—21 (September 1908) to +38 (July 1913)
14. Rate on 60-90 Day Paper	—31 (December 1908) to +45 (June 1913) and +53 (December 1907)

* Not corrected for seasonal variation.

use of bases increasing with time, that the normal growth, or "secular trend," is eliminated.¹

But there is another type of fluctuation which must be removed before the figures may be used as indices of business conditions. We must remove from the data those variations which are purely seasonal in character, which recur year after year as a consequence of the round of the seasons. If we could assume that the conditions of all the months of the year are alike, that there is no characteristic and recurrent variation which regularly differentiates the item for March from that for February or December, then we could legitimately compare the indices 95, 57, and 117. But the iron industry is seasonal in character. Just as a normal production for 1904 is not normal for 1918, so, within any year, a "normal" production for March is not normal for February or December. If we represent the average monthly production for any year by 100, the production for March, February, and December would be represented by 106, 94, and 100, respectively. Consequently, the seasonal indices 106, 94, and 100 should be subtracted from the indices of 95, 57, and 117 found above, in order to get significant figures for comparison. The differences are —11, —37, and +17.² These differences are "percentage deviations of original items from secular trend corrected for seasonal variation." They are comparable with each other; and they measure the relative activity (with respect to existing equipment) of the iron industry in March 1904, February 1908, and December 1918. The deviations reveal the depression in March 1904, the deep depression of February 1908, and the great activity of December 1918. These deviations become still more significant when we compare them with others computed in the

same manner for the period 1903-18. The months of deepest depression in the period were December 1903 (—44), January 1904 (—38), January to June 1908 (ranging from —37 to —44), November and December 1914 (—40 and —41). The months of greatest activity were March 1905 to October 1907 (ranging from +9 to a maximum of +26 in July 1907), August 1909 to April 1910 (ranging from +12 to +27), and September 1915 to December 1918 (ranging, with five exceptions, from +10 to +24). Negative deviations of —14 and —13 occurred in January and February 1918, as exceptional items in a series of forty positive deviations. They were undoubtedly caused by the severity of the winter, the railroad tie-up, and the fuel difficulties of the months in question. December 1917 (+2) and March 1918 (+6) were also low.

Other series of business statistics were treated in a fashion similar to that just explained for pig-iron production. For instance, the "percentage deviations of original items from secular trend corrected for seasonal variation" of the monthly rate of interest on sixty-to-ninety day commercial paper in New York for March 1904, February 1908, and December 1918 were —6, +12, and +39, respectively. The range of the deviations for the period 1903-18 was from —36 (November 1915) to +61 (June 1918); previous to the war period the deviations ranged from minima of —29 in August 1904 and September 1908 and —31 in December 1908, to maxima of +53 in December 1907 and +45 in June 1913. The range of the "percentage deviations of original items from secular trend corrected for seasonal variation" of twelve series for the period January 1903-July 1914 is given in Table B.

The ranges of the deviations of the series just given are, respectively: 85, 71, 36, 18.2, 58, 99, 32, 199, 61, 15.4, 59, and 84 points. With this variety of range from 15.4 to 199 points in fluctuations of the percentage deviations, it is clear that one per cent deviation does not indicate the same degree of fluctuation in all series. The percentage deviations of a *given* series are comparable with each other but a new unit must be found to secure comparability of the deviations of one series with those of another series. The proper selection of such a unit is the problem at hand.

¹ See Memoranda, p. 210 for a derivation of the slope of the line of secular trend, based upon the concept of arithmetic average increments between various pairs of years entering the period in question.

² We might, of course, compute ratios, instead of differences, and then take deviations from 100. The resulting figures would be

$$\frac{95}{106} - 100, \frac{57}{94} - 100, \text{ and } \frac{117}{100} - 100, \text{ or } -10, -39, \text{ and } +17,$$

respectively. It is obvious that except in extreme cases the deviations do not differ appreciably from those secured by the easier arithmetic process. The arithmetic process slightly tones down the extreme numbers, which is probably desirable.

TABLE C. — PERCENTAGE DISTRIBUTION OF THE CYCLES OF VARIOUS MONTHLY SERIES WITH RESPECT TO MULTIPLES OF THEIR STANDARD DEVIATIONS, 1903-16

SERIES	Number of Items	Standard Deviation	PERCENTAGE DISTRIBUTION				
			Between +1 and -1	Between +2 and -2	Between +3 and -3	Over +3	Under -3
1. New York Clearings	168	20.3	72	95	99	1	0
2. Production of Pig Iron	168	19.15	71	95	100	0	0
3. Outside Clearings	168	8.62	77	95	98	2	0
4. Bradstreet's Prices *	144	3.68	70	96	100	0	0
5. Imports	168	11.91	75	94	99	1	0
6. Building Permits	168	20.4	66	98	99	1	0
7. Railroad Gross Earnings	168	6.07	70	96	100	0	0
8. Shares Traded	163	49.6	78	96	98	2	0
9. Unfilled Orders U. S. S. C. †	56	32.3	62	96	100	0	0
11. Business Failures (Bradstreet)	168	13.55	72	96	99	1	0
12. Rate on 10 Railroad Bonds	168	2.82	69	98	99	1	0
13. Rate on 4-6 Months Paper	162	16.46	67	96	100	0	0
14. Rate on 60-90 Day Paper	168	19.66	69	96	100	0	0

* Period 1903-14.

† Quarterly figures.

Let us retrace the steps leading to this problem of obtaining a unit to secure comparability. The individual adjusted series are presumably corrected for secular trend and seasonal variation. The residual constituents of the original composite items are, then, the irregular and cyclical fluctuations. In the process of adjustment the original items were expressed as percentages of the corresponding ordinates of secular trend. Consequently, the corrected items of a given series, being relatives, are comparable over a period of years; that is, account has been taken of a change in the *level* of fluctuation. When we come to compare, however, the residual items, or "percentage deviations of the original items from the corresponding ordinates of secular trend corrected for seasonal variation," of two or more series with each other, a difficulty is met. The percentages do not mean the same thing in different series; they are not comparable. Some series fluctuate more violently than others; the items (or variates) of different series are more or less variable.

In order, then, to make a legitimate comparison of the fluctuations of the various series with each other, such series must be expressed in terms of proper units. The units must be selected with respect to the problem in hand and, therefore, must depend upon the variability of the series. What is needed is some common measure of variability, in terms of which each of the series may be stated. Such a measure is the "standard deviation" — the most widely used index of varia-

bility. The standard deviation is an average depending upon all of the items and varying directly with the dispersion or scatter of the items; the wider the distribution of the items the greater is the standard deviation.¹

In comparing the fluctuations of the several corrected series, therefore, we have expressed the percentage deviations in terms of the respective standard deviations as units.²

The figures thus obtained are termed "cycles." These cycles, whether in numerical or graphic form, are the appropriate magnitudes to compare or correlate. Charts of the cycles are given in Part IV on pp. 199-205 following. It will be noticed that for the period 1903-16 there are very few cases in which the cycles are more than +3.0 or less than -3.0. Table C shows the percentage distribution of the cycles of thirteen series having in all 2037 items. From two-thirds to three-fourths of the items of each series range between + σ and - σ , the standard deviations; from 94 to 98 per cent of the items of each series range between +2 σ and -2 σ ; from 98 to 100 per cent of the items of each series range between +3 σ and -3 σ ; only nine out of the total of 2037 items are over +3 σ , and none are under -3 σ . It is clear that the use of the standard deviation as a unit secures numerical comparability.

To the twelve series of cycles published in the January issue of the REVIEW there have been added eight new series. The numerical values of the monthly cycles of all twenty series are given in Part IV, 188-196.

¹ The standard deviation is the square root of the arithmetic mean of the squares of all deviations, deviations being measured from the arithmetic mean of the observations. If x represents deviations from the arithmetic mean, n the number of items, σ the standard deviation,

and Σ the process of summation, we have $\sigma^2 = \frac{1}{n} \Sigma (x^2)$.

"Since a small value for the standard deviation can arise only when the variates are closely concentrated about the mean or mode, and since a large value must be due to a relatively high frequency of the variates near the extremes of the distribution, the standard deviation

is a measure of the dispersion of the data. Because the effect of squaring is to diminish the importance of the smaller values and to exaggerate the importance of the larger values, a small value for the standard deviation shows conclusively that the data is (*sic*) highly true to type and stable, while on the other hand a large value may to some extent be due to the presence of the larger frequencies of the extreme variates and hence not altogether significant. But even with this qualification the standard deviation is a thoroughly practicable and reliable index of the dispersion of the data." (West, *Introduction to Mathematical Statistics*, p. 51.)

² See THE REVIEW OF ECONOMIC STATISTICS, January 1919, p. 36.

All the other data connected with the eight new series are likewise given in Part III, pp. 161-182.

The problem now before us is the measurement of the correlation between the cycles of the various series.

2. THE METHOD ADOPTED OF MEASURING CORRELATION BETWEEN THE CORRECTED SERIES, ILLUSTRATED BY PIG-IRON PRODUCTION AND INTEREST RATE ON SIXTY-TO-NINETY DAY COMMERCIAL PAPER

The corrected series which we have termed cycles are expressed in proper units for comparison. What method of measuring the correspondence or lack of correspondence between the fluctuations shall we adopt? To put the question with reference to a specified pair of series, what is the degree of correlation between the

lation. The maximum degree of correlation appears to be, not between items concurrent in time (synchronous), but between the items of pig-iron production and those of interest rate for, say, six months later. As would be expected, the amount of lag necessary to secure the best fit of the two graphs to each other varies. The graph of pig-iron production for 1904-06 is closely similar to the graph for interest rate for a year later, that is, 1905-07. For this short period interest rate lags one year. During the period 1907-11 the cycles of interest rate lag six months behind pig-iron production. During the period January 1912-July 1914 the lag of interest rate for maximum correlation with pig-iron production appears to be three or four months. Comparison of the two graphs with the cycles of interest rate shifted to the

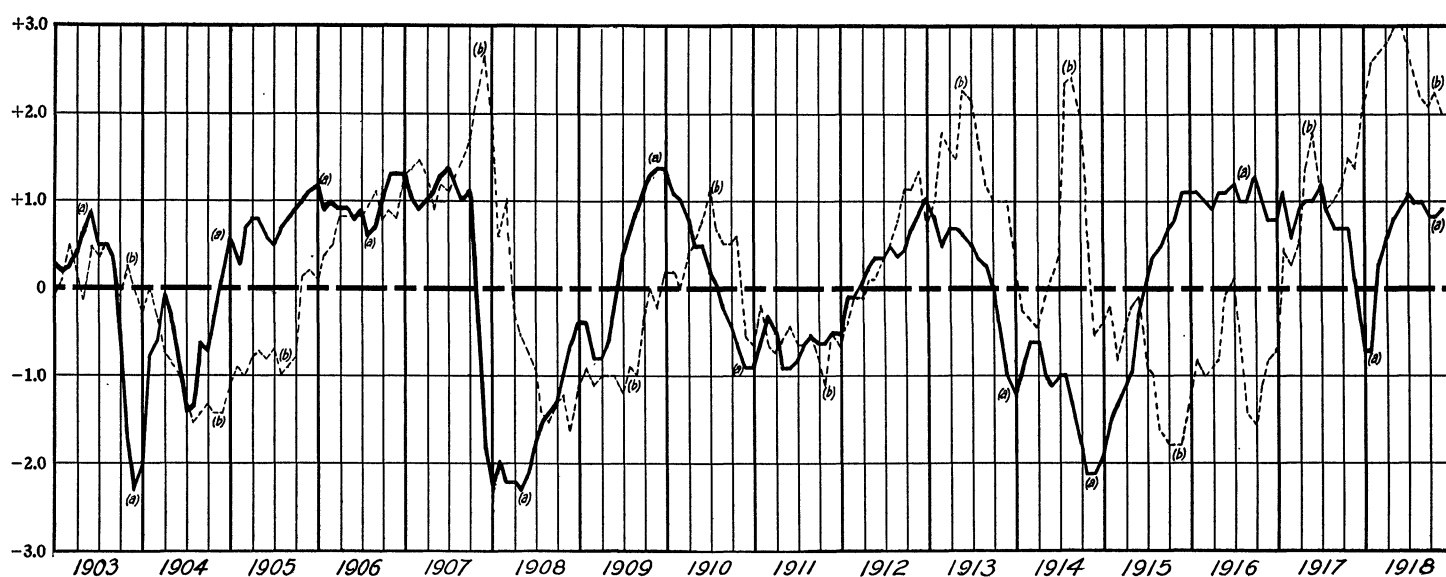


CHART A. — Comparison of cycles of (a) monthly tonnage of the production of pig iron in the United States, and (b) monthly rate of interest on sixty-to-ninety day commercial paper in New York, 1903-18.

cyclical fluctuations of, say, the monthly tonnage of pig iron produced in the United States (Chart 102, p. 201) and the monthly rate of interest on sixty-to-ninety day commercial paper in New York (Chart 114, p. 205)?

The obvious procedure is to plot the graphs for the two series on the same chart, and examine such graphs for similarity or dissimilarity of movement. Chart A presents the cycles of pig-iron production and rate of interest on sixty-to-ninety day commercial paper. In comparing the two graphs let us limit ourselves for the present to the period ending with July 1914.

Upon examining Chart A the first feature to attract notice is the similarity of the wave movements. It is evident that the cycles correspond, or in other words, that they are positively correlated. On the whole, positive items of one series correspond to positive items of the other series and negative items of one series correspond to negative items of the other series.

A second feature of Chart A that commands immediate notice is that the wave movements of the rate of interest on sixty-to-ninety day commercial paper follow or lag behind the wave movements of the tonnage of pig iron produced in the United States. This fact of lag, and it appears to be a fact, may be stated in terms of corre-

lation. The maximum degree of correlation appears to be, not between items concurrent in time (synchronous), but between the items of pig-iron production and those of interest rate for, say, six months later.

The systematic correspondence of the two wave movements, whether the lag of interest rate be one year, six months, three months, or any other interval, was definitely interrupted in August 1914. Immediately following the declaration of war interest rate rose violently and pig-iron production slumped. Interest rate on sixty-to-ninety day paper then fell to a low level in 1915-16 and recovered in 1917-18. Pig-iron production rose violently during 1915 and remained high during 1916-18, except for the slump in the winter of 1917-18, when there was a combination of intensely cold weather, fuelless days and railroad congestion.

From the comparison which has just been made it would appear that in peace times movements in pig-iron production anticipate movements in interest rate on commercial paper by an interval of three to twelve months. In order to arrive at this conclusion from a comparison of the graphs, it is necessary to make, not merely one comparison of pairs of items for the same dates, but many comparisons, with items of one series paired with various items of the other series.

To secure such comparisons a mechanical contri-

vance was devised. The cycle charts were drawn on translucent tracing cloth. A box was constructed with a glazed top illuminated by electric lights placed within the box. The accompanying figure illustrates the device.

A table showing a record of the observations for all series is given in Part IV, p. 184. The results of the comparison of the cycles of pig-iron production and interest rate on sixty-to-ninety day commercial paper are as follows:

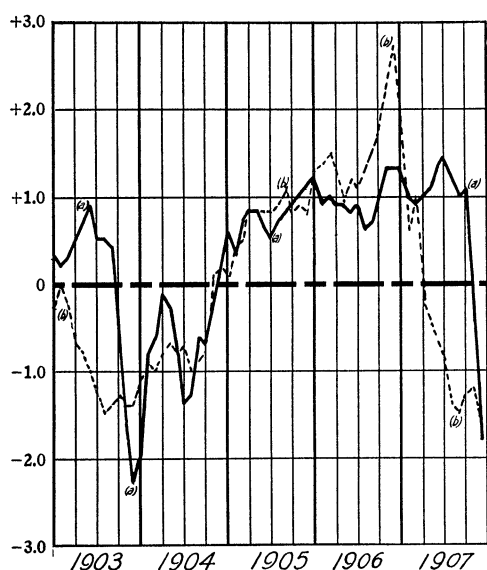


CHART B.—Comparison of cycles of (a) monthly pig-iron production and (b) monthly interest rate of one year later, 1903-07 (dates refer to pig-iron production).

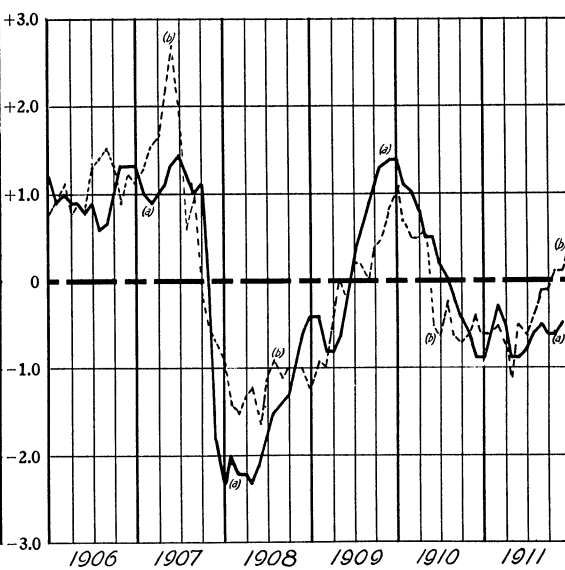


CHART C.—Comparison of cycles of (a) monthly pig-iron production and (b) monthly interest rate of six months later, 1906-11 (dates refer to pig-iron production).

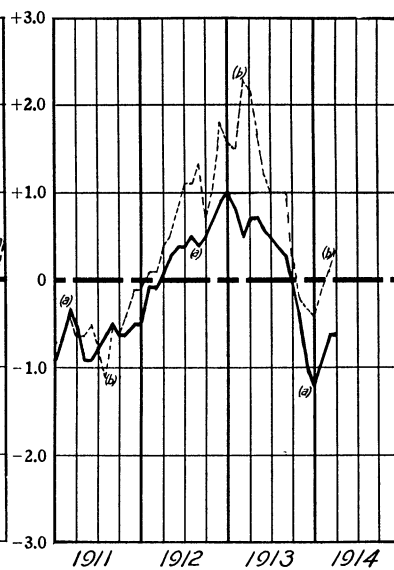


CHART D.—Comparison of cycles of (a) monthly pig-iron production and (b) monthly interest rate of three months later, Jan. 1911-April 1914 (dates refer to pig-iron production).

One cycle chart was placed upon another so that the horizontal zero lines and the vertical year lines coincided. The two graphs were then in position to determine the correspondence in sign and magnitude of concurrent items. A shift of the upper chart to the right or left enabled the observer to estimate the relative positions of the two graphs which would secure the best fit of one curve to the other.

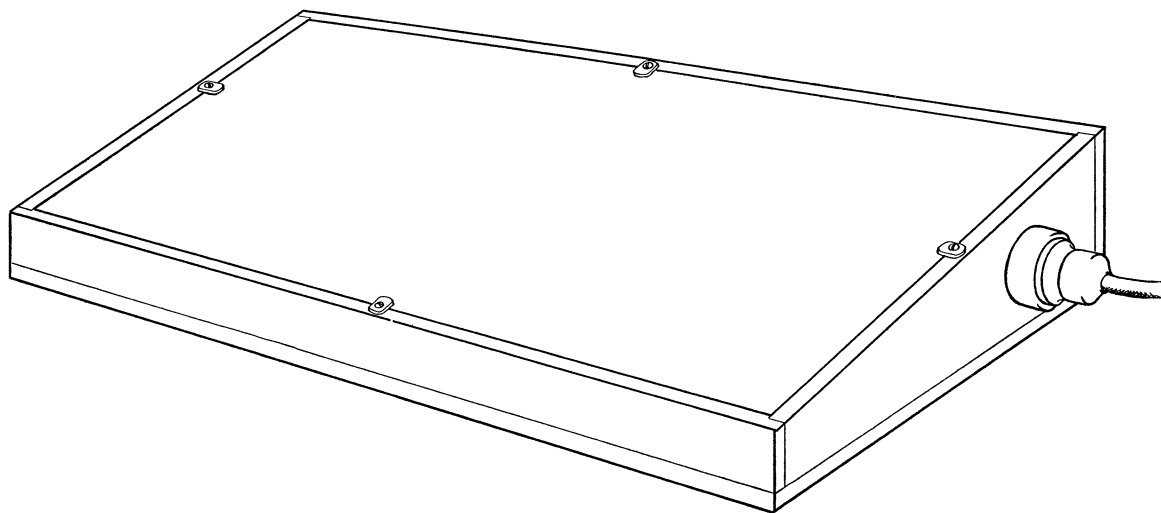
Twenty cycle charts were compared in this way, making 190 comparisons by each of three independent observers. Each observer recorded his results as follows:

- (a) Degree and sign of correlation: high, moderate, low; + or -.

COMPARISON OF CYCLES, 1903-14

2. Pig-iron production of the United States	} compared with	14. Interest rate on sixty-to-ninety day commercial paper.
(a) Correlation positive and high.		(c) Fairly systematic for lag of, say, 6 months in no. 14, but varies from 3 to 12 months.
(b) No. 14 lags 5 or 6 months for maximum correlation with no. 2.		

The graphic method of the measurement of correlation has advantages; but it also has limitations. It is reliable when the correspondence between the curves is simple and regular. It is uncertain when the correlation and lag are variable. The method is easily applied and it yields valuable results. Those results, however,



Illuminated Box with Glazed Top to Facilitate Comparison of Cycle Charts.

- (b) Direction and extent of lag in months for the maximum correlation during the entire period.
(c) Characterization of the consistency of lag for maximum correlation: excellent, good, fair, poor.

are approximations dependent upon the judgment of the observer. His personal equation, preconceived notions, or theoretical bias influence his conclusions. Such conclusions need to be checked up by a more

TABLE D.—CYCLES OF (a) MONTHLY TONNAGE OF PIG IRON PRODUCED IN THE UNITED STATES AND (b) MONTHLY INTEREST RATE ON 60-TO-90 DAY COMMERCIAL PAPER IN NEW YORK

	1903		1904		1905		1906		1907		1908		1909		1910	
	(a)	(b)	(a)	(b)	(a)	(b)	(a)	(b)	(a)	(b)	(a)	(b)	(a)	(b)	(a)	(b)
January	+ .3	— .1	—2.0	— .3	+ .6	—1.1	+1.2	+ .1	+1.3	+1.3	—2.3	+1.9	— .4	—1.1	+1.4	+ .2
February	+ .2	+ .1	— .8	.0	+ .3	— .9	+ .9	+ .4	+1.0	+1.4	—2.0	+ .6	— .4	— .9	+1.1	+ .2
March	+ .3	+ .5	— .6	— .3	+ .7	—1.0	+1.0	+ .5	+ .9	+1.5	—2.2	+1.0	— .8	—1.1	+1.0	.0
April	+ .5	+ .2	— .1	— .7	+ .8	— .8	+ .9	+ .8	+1.0	+1.3	—2.2	— .2	— .8	—1.0	+ .8	+ .4
May	+ .7	— .1	— .3	— .8	+ .8	— .7	+ .9	+ .8	+1.1	+ .9	—2.3	— .5	— .6	—1.0	+ .5	+ .5
June	+ .9	+ .5	— .8	—1.0	+ .6	— .8	+ .8	+ .8	+1.3	+1.2	—2.1	— .7	— .2	—1.0	+ .5	+ .8
July	+ .5	+ .4	—1.4	—1.3	+ .5	— .7	+ .9	+ .8	+1.4	+1.1	—1.8	— .9	+ .4	—1.2	+ .2	+1.1
August	+ .5	+ .5	—1.3	—1.5	+ .7	—1.0	+ .6	+ .9	+1.2	+1.3	—1.5	—1.4	+ .6	— .9	.0	+ .7
September	+ .4	+ .3	— .6	—1.4	+ .8	— .9	+ .7	+1.1	+1.0	+1.5	—1.4	—1.5	+ .9	—1.0	— .2	+ .5
October	— .5	— .1	— .7	—1.3	+ .9	— .8	+1.0	+ .8	+1.1	+1.7	—1.3	—1.3	+1.1	— .3	— .4	+ .5
November	—1.6	+ .3	— .3	—1.4	+1.0	+ .1	+1.3	+ .9	— .1	+2.2	—1.0	—1.2	+1.3	.0	— .6	+ .6
December	—2.3	.0	+ .1	—1.4	+1.1	+ .2	+1.3	+ .8	—1.8	+2.7	— .6	—1.6	+1.4	— .2	— .9	— .5

	1911		1912		1913		1914		1915		1916		1917		1918	
	(a)	(b)	(a)	(b)	(a)	(b)	(a)	(b)	(a)	(b)	(a)	(b)	(a)	(b)	(a)	(b)
January	— .9	— .6	— .5	— .6	+1.0	+ .7	—1.2	+ .3	—1.9	— .4	+1.1	—1.2	+ .8	— .6	— .7	+2.1
February	— .6	— .2	— .1	— .4	+ .8	+1.0	— .9	— .2	—1.5	— .2	+1.1	— .8	+1.1	+ .5	— .7	+2.6
March	— .3	— .6	— .1	— .1	+ .5	+1.8	— .6	— .3	—1.3	— .8	+1.0	—1.0	+ .6	+ .3	+ .3	+2.7
April	— .5	— .7	+ .1	— .1	+ .7	+1.6	— .6	— .4	—1.1	— .4	+ .9	— .9	+ .9	+ .6	+ .6	+2.8
May	— .9	— .6	+ .3	+ .1	+ .7	+1.5	—1.0	— .1	— .9	— .2	+1.1	— .8	+1.0	+1.4	+ .8	+2.9
June	— .9	— .4	+ .4	+ .1	+ .6	+2.3	—1.1	+ .1	— .3	— .1	+1.1	— .1	+1.0	+1.8	+ .9	+3.1
July	— .8	— .6	+ .4	+ .4	+ .5	+2.2	—1.0	+ .4	+ .1	— .9	+1.2	+ .1	+1.2	+1.1	+1.1	+2.8
August	— .6	— .6	+ .5	+ .5	+ .4	+1.7	—1.0	+2.3	+ .4	—1.0	+1.0	— .6	+ .9	+ .9	+1.0	+2.5
September	— .5	— .5	+ .4	+ .8	+ .3	+1.2	—1.3	+2.4	+ .5	—1.6	+1.0	—1.4	+ .7	+1.0	+1.0	+2.2
October	— .6	— .8	+ .5	+1.1	.0	+1.0	—1.8	+2.0	+ .7	—1.8	+1.3	—1.5	+ .7	+1.2	+ .8	+2.1
November	— .6	—1.1	+ .7	+1.1	— .4	+1.0	—2.1	+1.1	+ .8	—1.8	+1.1	—1.1	+ .7	+1.5	+ .8	+2.3
December	— .5	— .5	+ .9	+1.3	—1.0	+1.0	—2.1	— .5	+1.1	—1.8	+ .8	— .8	+ .1	+1.4	+ .9	+2.0

objective method of measuring correlation. There is need of a quantitative measure, of a coefficient of correlation, which will enable us to decide, for instance, whether the correlation between pig-iron production and interest rate is greater or less than the correlation between wholesale prices and interest rate. There is need of such an average or index, moreover, to tell us what lag to take, *on the whole*, when examination reveals a variable lag. In other words, the inspection of the graphs with the aid of the illuminated box is an excellent preliminary operation. The results obtained give a valuable guide for further investigation; but a *quantitative* measure of correlation is needed to supplement and test the tentative conclusions which the graphic method has provided.

To secure the quantitative measure desired it is necessary to define correlation in quantitative terms. In the first place, correlation between two series may be either direct (positive) or inverse (negative). Direct correlation exists when positive items of the first series are associated with positive items of the second series, and negative items of the first series are associated with negative items of the second series. In the second place, correlation between two series may be numerically high or low. Correlation is high if large items of the first series are associated with large items of the second series, and small items of the first series are associated with small items of the second series. Correlation is low if

large items of the first series are associated with small items of the second series and *vice versa*. Correlation is zero if items of the first series, negative and positive, large and small, are indiscriminately associated with the various items of the second series. For convenience in referring to the two series the first will be called the *standard* series and the second will be called the *correlative* series.¹

In attempting to obtain a numerical measure of correlation in accordance with the concept just defined, we have as our material the cycles, or in other words, "the percentage deviations of original items from secular trend corrected for seasonal variation" expressed in comparable units, the respective standard deviations.

Let us consider the numerical values of the cycles of

¹ The definition of correlation given by A. L. Bowley (*Elements of Statistics*, p. 316) is as follows: "When two quantities are so related that the fluctuations in one are in sympathy with the fluctuations of the other, so that an increase or decrease of one is found in connection with an increase or decrease (or inversely) of the others, and the greater the magnitude of the changes in the one, the greater the magnitude of the changes in the other, the quantities are said to be *correlated*." This is really a definition of correlation for the *first differences* (changes in adjacent items) of one series and the associated *first differences* of another series. In the present discussion we are interested in the correlation between *cycles*, *not* in the correlation between *first differences of cycles*. Generally, if the correlation is high for cycles it will be high for first differences; but the exceptions are important. For instance, two series may have opposite month-to-month oscillations, these oscillations being superimposed upon like cyclical movements. In such case, the correlation between first differences would be inverse and between cycles would be direct.

pig-iron production and the corresponding values, either concurrent or with a definite lag, of the cycles of the interest rate on sixty-to-ninety day commercial paper in New York. What items shall we pair? The preliminary graphic comparison indicated that the amount of lag of interest rate ranged, at different periods, from three months to a year; maximum correlation for the whole period January 1903–July 1914 appeared for a lag of six months. Accordingly, we will set up a one-to-one correspondence between the items of pig-iron production and those of interest rate six months later. The items for the two series are given in Table D.

Beginning with January 1903 for pig-iron production and July 1903 for interest rate, consider the pairs of items: (+.3, +.4), (+.2, +.5), (+.3, +.3), (+.5, −.1), (+.7, +.3), (+.9, 0), (+.5, −.3), (+.5, 0), . . . , (−1.2, +.4). The last two items are for January and July 1914, respectively.

Consider the products of the paired items, +.12, +.10, +.09, −.05, +.21, 0, −.15, 0, . . . , −.48, respectively. Positive products occur if the factors are of like algebraic sign, negative products occur if the factors are of unlike signs. Moreover, the products vary even though the sum of the factors is constant. Thus, the sums of the first two pairs are each +.7 but the products are +.12 and +.10, respectively. The larger product results from factors nearly equal in size, rather than from a large and a small factor. This statement may be generalized. The maximum product of two factors having a constant sum occurs when the factors are equal.¹

Let us consider the properties of the products of pairs of items with respect to the problem of finding a coefficient of correlation. Suppose we start out with the items of a certain series, pig-iron production, as the given data. What series can we build up or construct artificially which possesses the maximum degree of correlation with the given series? It is obvious that a series constructed with items identical with those of pig-iron production is most highly correlated with it. Such a series would, for a given total to distribute as items, also give maximum values for all the products of corresponding pairs.² Furthermore, all products would be positive. On the other hand, if we should build up a series by merely changing the signs of the items of pig-iron production and pair the items of the two series, the products would be maxima numerically, but all negative in sign. Thought of graphically, the curve presenting the series so constructed would be the reflection of the former curve in a horizontal mirror. The *inverse* correlation would be a maximum in this case. Between the two extreme cases there could be constructed various series having various degrees of correlation with pig-iron production. By varying the magnitudes, or the signs, or both, a series could be devised more or less similar to a given series.

It is clear, then, that the greater the *sum* of the products of corresponding items, the greater will be the correlation between the two series, the maximum sum

of products occurring when equal items are paired, that is, when the two series are identical. The *sum of products of corresponding pairs of items* may, therefore, be used as a measure of correlation. In order that one value, however, computed from, say, 50 items, may legitimately be compared with another value, computed from another number of items, say 100, it appears best to divide the sum of the products by their number. *That is to say, we shall use as our coefficient of correlation the arithmetic mean of the products of corresponding pairs of cycles.*³

The coefficient cannot have a numerical value greater than unity; it takes that value when the two series paired have corresponding cycles of like numerical values. In such case the coefficient will be +1 when the algebraic signs of the cycles paired are alike; it will be −1 when the algebraic signs are unlike.⁴ In all other cases the coefficient will take numerical values ranging

¹ Stated in geometric terms: the square is the rectangle of maximum area for a given perimeter. The proof is simple. Given a to find the value of x which gives a maximum value of $x(a - x)$. Differentiate with respect to x , equate to zero and solve, getting $x = \frac{a}{2}$.

² The reader may need to be reminded that both series are supposed to be in terms of their standard deviations. That is, the sum of the squares of the cycles for all the series is the same. The absolute sums of a like number of the items for all series in such case does not vary widely, the range being from 124 to 146. For instance, the absolute and algebraic sums of the cycles of various series for the period 1903–16 is given by the following table:

SERIES	SUM OF CYCLES, 1903–16		Arithmetic Average
	Absolute	Algebraic	
1. New York Clearings	133.1	+ .5	+ .003
2. Production of Pig Iron	145.9	+ .5	+ .003
3. Outside Clearings	124.4	− 1.2	− .007
4. Bradstreet's Prices *	113.2	+ 3.4	+ .024
5. Imports	125.2	+ 2.6	+ .016
6. Building Permits	132.9	+ .5	+ .003
7. Railroad Gross Earnings	134.7	+ 2.9	+ .017
8. Shares Traded	122.3	+ 6.5	+ .040
9. Unfilled Orders U. S. S. C.	48.6 †	− 1.6	− .029
10. Tonnage Entered	‡	‡	‡
11. Business Failures (Bradstreet)	130.6	+ 4.8	+ .029
12. Yield of 10 Railroad Bonds	137.6	− .2	− .001
13. Rate on Four-to-six Months Paper	134.2	+ 2.0	+ .012
14. Rate on Sixty-to-ninety Day Paper	141.0	− 1.0	− .006
15. Rate on Call Loans — Mitchell	‡	‡	‡

* 1903–14 for (4) Bradstreet's Prices. † Quarterly cycles. ‡ Cycles not computed.

³ Since the cycles are deviations from the linear secular trend, so that their algebraic sum is zero, and since the standard deviation of each series is unity, the coefficient of correlation above defined is the well-known Pearsonian coefficient. It was invented by Bravais in 1846 and developed and applied by Galton, Pearson, Edgeworth, Yule, and others. See the section of this paper entitled "General Considerations Relating to the Measurement of Correlation," pp. 130, for a brief history of the coefficient with reference to important memoirs.

⁴ If x and y represent the deviations of the items of two time series from their respective linear secular trends, n stands for the number of pairs, r represents the coefficient of correlation, and Σ stands for "the sum of such terms as" we have:

$$r = \frac{1}{n} \Sigma \left(\frac{x}{\sqrt{\Sigma x^2}} \cdot \frac{y}{\sqrt{\Sigma y^2}} \right) = \frac{\Sigma xy}{\sqrt{\Sigma x^2 \cdot \Sigma y^2}}.$$

For maximum positive correlation $x = y$ and r , therefore, becomes equal to +1. For maximum negative correlation $x = -y$ and $r = -1$. Consequently, the limits of possible values of r are +1 and −1. See Bowley, *Elements of Statistics*, p. 319, for a rigorous proof that $-1 \leq r \leq +1$.

between 0 and 1. The zero of the scale occurs for the case in which the pairing of the items of any two series is made at random. That is, if we should pair a given series, say, the production of pig iron, with a series of cycles constructed by taking as its items the numbers of the pages of a book opened at random, we should expect the coefficient of correlation to be approximately zero. Or, to cite another illustration of the meaning of the zero of the correlation scale, suppose that the numerical values of the monthly cycles of the production of pig iron are written on separate slips of white paper and deposited in a hat. Suppose, further, that the numerical values of the monthly cycles of the interest rate on sixty-to-ninety day commercial paper be written on separate slips of yellow paper and deposited in another hat. Now suppose that a white and a yellow slip are drawn simultaneously from the two hats and that the product of the two numbers drawn is taken;

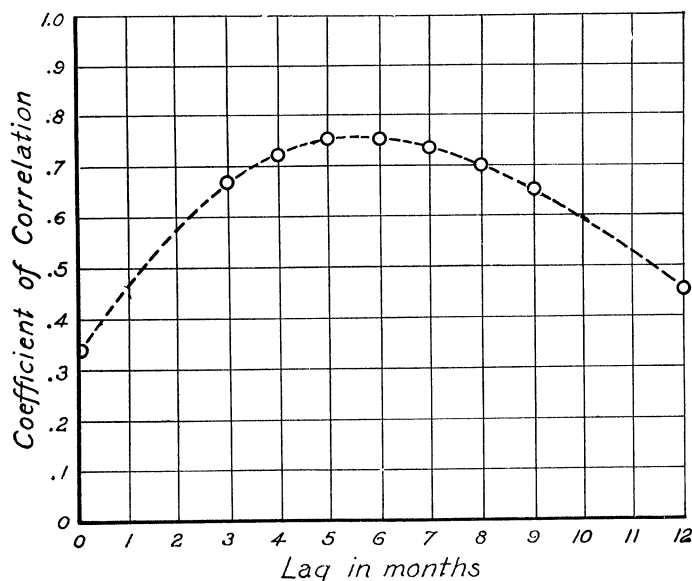


CHART E. — Coefficients of correlation between cycles of pig-iron production and interest rate on sixty-to-ninety day commercial paper, with 0, 3, 4, 5, 6, 7, 8, 9, and 12 months lag of interest rate. (The maximum correlation, $r = +.75$, is for a lag of five or six months.)

the process being continued until all the slips are thus drawn and the products of each pair taken. From the random character of the selection, we should expect some products to be positive, some negative; further, we should expect an approximately equal number of positive and negative products. In other words, we should expect the arithmetic mean of the products, i. e., the coefficient of correlation, to be numerically small, approximately zero.¹

Let us now return to the question, what is the degree of correlation between the cycles of pig-iron production and those of interest rate on sixty-to-ninety day commercial paper six months later? The arithmetic average of the products of corresponding items is $+.75$. We shall adopt $r_{0,6}$ as the symbol to indicate the coefficient of correlation in question. The subscripts indicate that the items of the former series are paired with items of the latter series six months later.

The limiting values of the scale of correlation are then $+1$, 0, and -1 . The first value, $+1$, indicates that the two series of cycles are identical; the second

value, 0, indicates a complete lack of correspondence between the series, such as would occur from pairing items at random; the third value, -1 , indicates that the items of the two series are numerically identical but opposite in sign. What do intermediate values indicate? In other words, what is the meaning of such coefficients as $+.04$, $+.37$, $+.48$, $+.80$, $+.96$ or their negatives? It is clear from what has been said that these coefficients indicate that the pairs of items yielding them have not been selected at random; nor are they identical. As it happens, the coefficients named have been computed from pairs of biometric items. The first, $+.04$, is the coefficient of correlation for breadth of head and ability of human beings. Being approximately zero it indicates that the pairing in nature occurs at random. The remaining coefficients indicate more or less system in the pairing, the degree of system or correspondence being indicated by the size of the coefficient. The coefficient $+.37$ is for length of forearm and stature; $+.48$ is for weight and stature; $+.80$ is for length of femur and stature; $+.96$ is for length of right and left femur.²

But is the correlation a maximum for a lag of six months in interest rate? Computing the coefficients of correlation between pairs with a lag in interest rate of 0, 3, 4, 5, 6, 7, 8, 9, and 12 months, we have the following results:

$r_{0,0}$	$r_{0,3}$	$r_{0,4}$	$r_{0,5}$	$r_{0,6}$	$r_{0,7}$	$r_{0,8}$	$r_{0,9}$	$r_{0,12}$
$+.34$	$+.67$	$+.72$	$+.75$	$+.75$	$+.73$	$+.70$	$+.65$	$+.45$

These coefficients indicate that the maximum correlation is for a lag of four-to-eight months in interest rate. The fit of the cycles of pig-iron production and those of interest rate six months later is shown in Chart E.

Having determined the maximum correlation between the two series, there remains to be considered one further question. Is $+.75$ a low or a high figure in the correlation scale? Does $+.75$ indicate a significant likeness between the two series compared or is it a likeness that might readily occur accidentally? The general question of the significance of the values of the correlation scale has been considered non-technically in the preceding pages. To answer our specific questions fully and rigorously would lead us into the intricacies of the mathematics of the theory of probability, an unnecessary digression.³ Some further consideration of the general question, however, is advisable. Greater

¹ We may remind the reader that we are dealing with series of cycles which have an approximately equal number of positive and negative items. Further, the frequency of small cycles is greater than the frequency of large cycles, so that we expect a larger number of small products than of large products. In other words, given a normal distribution of the items of each series, there will follow a normal distribution of the products of random pairs.

² The meaning of the various values in the correlation scale, as of those in any scale, the centigrade scale of temperatures, for instance, can only be "sensed" by repeated experience.

³ For a mathematical discussion of the theory of probability see, for instance, Arne Fisher, *The Mathematical Theory of Probabilities*, vol. i (1915), "Mathematical Probabilities and Homogeneous Statistics," and L. D. Weld, *Theory of Errors and Least Squares* (1916).

precision must be given to the concept of "expectation," a concept fundamental to a proper understanding of the significance of the correlation scale. It is clear that a small amount of correlation and a small coefficient might be *expected* to occur between two series chosen quite at random. It is also manifest that although a high coefficient for two series chosen at random is possible, it is not probable. Expressed otherwise, the larger the coefficient of correlation the smaller is the probability that it has resulted accidentally.

The probability that a *given* coefficient, say $+.60$ or $+.65$, may occur for two series chosen at random does not depend exclusively upon the size of the coefficient. It depends as well upon the number of pairs of items which have been used in computing the coefficient. For instance, a coefficient based upon 12 pairs does not carry the same weight, it is not so significant of persistent relationship as one based upon a larger number of pairs, say 130. Thus, the coefficient of correlation for monthly pig-iron production for the calendar year 1904 and monthly interest rate on sixty-to-ninety day commercial paper for 1905, twelve months lag and 12 pairs of items, is $+.61$. The coefficient for the same series and nine months lag but for pairs of items between the dates January 1903 and July 1914, 130 pairs, is $+.65$. The latter coefficient ($+.65$) is much more significant than the former ($+.61$), not because of the numerical difference between them, but because of the wider basis of the coefficient (130 pairs as compared with 12). Using a twelve months lag for the period January 1903 to July 1914, 127 pairs, the coefficient is $+.45$. This result, considered in connection with the result of $+.65$ previously obtained, means that a twelve months lag is not nearly so persistent during the entire period under discussion as a nine months lag.

In judging the significance of coefficients, then, the fundamental notions are the size of the coefficient and the number of pairs of items utilized. Expressed in mathematical terms, the probability that a given coefficient has resulted from totally unconnected causes is a function of the coefficient and the number of pairs of items used in its computation. The form of this function has been derived. The value of the function which is most widely used is called the "probable error."¹ The principles underlying the notion of probable error are the same as those underlying games of chance. Suppose we are asked, if a coin be tossed what is the probability that it will fall heads? The answer comes readily, one chance out of two. Our knowledge of the shape of coins and our experience with gravitation give us confidence in our prediction. The "subjective probability" is one-half. In other words, *if* the coin is circular, *if* it has two flat faces, *if* it has a narrow edge, *if* it is evenly weighted, *if* it is tossed without control,

and *if* gravitation operates as it has been observed to operate in the past, the probability that the coin will fall heads is one-half. Similarly, if we place seventy white balls and thirty black balls in a bowl, mix them thoroughly, and then draw one at random, the "subjective probability" of drawing a white ball is seven-tenths. The known conditions of the processes of tossing coins and drawing balls enable us to predict with confidence *before making experiments in the particular cases*.

It is not difficult to decide upon the probability of an occurrence of an event in such simple games of chance as we have just considered. But for complex economic problems the situation is quite different. The conditions of economic processes are not known, or at any rate they are not known accurately enough for us to decide upon the probability of an event depending upon them. It is possible to arrive at the probability of occurrence for an economic event only by use of the actual record of the happening of events in the economic world. We cannot set up an experiment in economics; we must draw our conclusions and state our probabilities on the basis of evidence that is yielded automatically, so to speak. To draw a parallel between economic events and the process of drawing black and white balls from a bowl, the problem of the determination of a "probability" in economics is similar to the problem of ascertaining the ratio between the unknown numbers of black and white balls in a bowl based upon a record of sample drawings of, say, ten balls at a time. In other words, just as we may estimate the relative number of black and white balls in a bowl from a record of experiments, so we may determine an "objective probability" from available economic statistics.

To return to the illustration of coin tossing: when we say, "the probability is one-half that the coin will fall heads," we do not mean that if we actually toss the coin 1,000 times we shall get exactly 500 heads and 500 tails. In this connection, we shall quote the explanation given by R. H. Hooker:²

"If we toss for a thousand times we shall almost certainly not get *exactly* 500 heads and 500 tails: we should get somewhere near those figures, and we should be very much surprised if we got 600 heads and only 400 tails. So great a difference as a proportion of 3 : 2 is very unlikely to occur, and if it did occur we should suspect very strongly that the tosser was able to exercise some (perhaps small) influence over the coin. Yet if we only toss five times, the nearest approach to equality that we can possibly get is 3 : 2; and if we toss ten times we are almost as likely to get 6 : 4 as 5 : 5. Thus a distinct inequality in the case of a small number of observations is of no significance, whereas it is important in the case of a large number. Similarly, a distinct correlation coefficient is no evidence of connection if the number of observations be small, but the same figure is if they be numerous. With a given coefficient, and a knowledge of the number of observations on which it is based, the probability of its being a true con-

¹ See "Regression, Heredity, and Panmixia," by Karl Pearson, in *Transactions of the Royal Society*, 187 (1896), A, p. 266, and 191 (1898), pp. 229-311, especially p. 242. See also, Yule's *Introduction to the Theory of Statistics*, 1st ed., p. 348.

² R. H. Hooker, "An Elementary Explanation of Correlation," *Quarterly Journal of the Royal Meteorological Society*, October 1918, pp. 285-286.

nection can always be calculated, and the 'probable error' has been found to be the most useful guide. In practice it has been found *absolutely* safe to say that there is certainly a connection between two phenomena if the correlation coefficient be at least six times its probable error. This represents odds of many thousands to one in favour of the connection being real. . . .

"Perhaps [an] example regarding probability may be cited as an illustration. It is well known that the bank at Monte Carlo 'always' wins, and that a very steady revenue is derived therefrom. Yet it would seem, to judge by the rules of the play, that the odds in favour of the bank are only very slight. Slight as they

chosen at random from a universe having, in general, the given degree of correlation.¹ In other words, suppose there are billions of pairs of items with a correlation coefficient of $+.60$. From these billions suppose we take at random 100 items and compute a coefficient. We then take other random samples of 100 pairs and compute the coefficients. These coefficients will not all be $+.60$, but will be scattered symmetrically about $+.60$ according to the "normal curve of error." That is, there will be an equal number of divergences above and below $+.60$, and the number of divergences in excess or defect will decrease as the size of the divergence increases. The shape of the normal curve of error

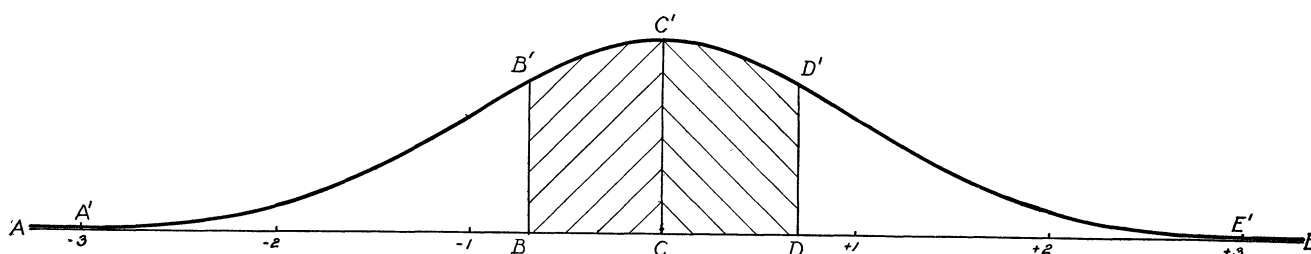


CHART F. — The curve $A'B'C'D'E'$ is the normal curve of error of a coefficient of correlation for samples of 100 pairs of items.

appear, however, they tell in the long run, owing to the large number of times the game is played. But still it is not really an absolute certainty; a particular individual may upon occasion 'break the bank,' and therefore it is quite conceivable that such an operation may occur so frequently that the bank will have lost by the end of the year. With a correlation coefficient six times its probable error, the chances of there being no connection are far more remote than those of a given individual breaking the bank at Monte Carlo.

"Thus, whatever the correlation coefficient (if not unity) there is always a remote chance — it may be many millions to one — that the connection is accidental. But it is always strictly true that the coefficient measures the resemblance that has actually held for the observations included in the calculations. It is only when we want to know whether the connection is causal, and, as a corollary, whether it will still hold in the future, that we have to consider questions of probability. And it may perhaps not be amiss to remark that when a writer points out, as frequently occurs, that 'inspection of the diagram shows that ups and downs correspond fairly, and therefore the two sets of phenomena are interdependent,' such a writer has frequently quite small odds in his favour."

A coefficient of correlation computed from a limited series of economic data has, then, a margin of error. Expressed in other terms, any given coefficient of correlation is an attempt to estimate, from a sample, relationship, and that estimate has a probable error. As we have said the probable error "is a function of the coefficient and the number of pairs of items used in its computation." Stated more precisely, the probable error of a given correlation coefficient is that divergence, on the average, on either side of the coefficient, within which exactly half the values of a large number of coefficients would lie if computed from pairs of items

is bell-like, as illustrated by the accompanying diagram giving the normal curve of error of a coefficient of correlation based upon 100 pairs of items (Chart F). Point D on the diagram corresponds to the coefficient of $+.64$ and point B to one of $+.56$.² The area bounded by any two ordinates, as CC' and DD' , the base line, CD , and the curve, $C'D'$, corresponds to the number of coefficients we may expect to get between $+.60$ and $+.64$. The area, $BB'D'DB$, is one-half of the area included between the curve $A'C'E'$ and the base line ACE . The distances CB and CD represent the probable error, that is, the chances are equal that the coefficient computed from a given sample of 100 items will fall between the values $+.56$ and $+.64$. Similarly, the probability is about 68 to 32, or two to one, that the coefficient of such a sample will fall between $+.54$ and $+.66$; it is 19 to 1 that the coefficient will fall between $+.47$ and $+.73$; it is 997 to 3 that the coefficient will fall between $+.41$ and $+.79$.

As has been said, the probable error depends on the size of the sample. For a coefficient of $+.60$ based upon 25 pairs of items the probable error is $.087$; for 1,000 pairs of items it is $.014$. That is, for a sample of 1,000 pairs taken from a universe having the coefficient of $+.60$ the chances are equal that the coefficient computed from the sample will be between $+.586$ and $+.614$. Chart G, drawn on the same scale as Chart F,

¹ The expression for the probable error of the coefficient of correlation, r , is $0.674 \frac{1-r^2}{\sqrt{n}}$, where n indicates the number of items paired.

In case the number of items is very large the probable error will be approximately zero. The word "universe" is used here to describe the complete homogeneous group of pairs.

² These values, $+.64$ and $+.56$, are obtained by substituting $r = +.60$ and $n = 100$ in the expression for the probable error, $0.674 \frac{1-r^2}{\sqrt{n}}$, finding the value $.043$, then adding and subtracting $.04$ to and from $+.60$. The coefficient and its probable error are usually written $+.60 \pm .04$.

shows the distribution of the coefficients of correlation for samples of 1,000 items. Comparison of the two charts shows that the dispersion of the coefficients com-

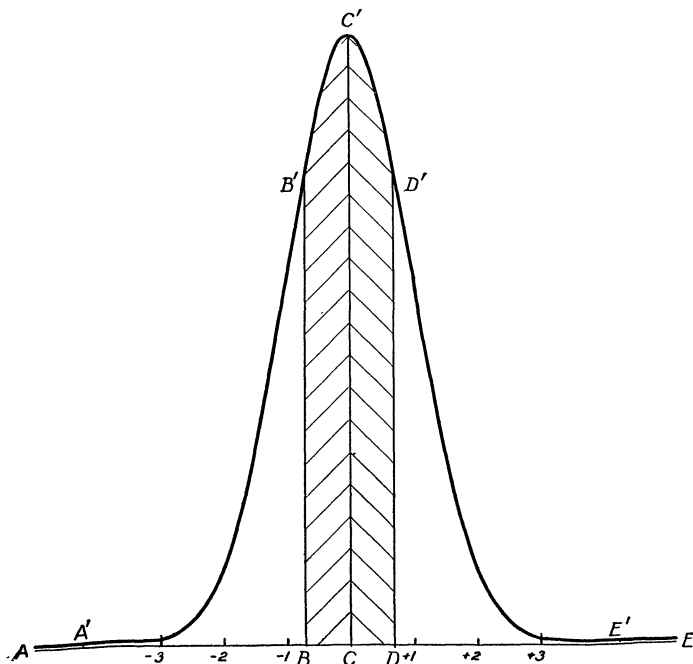


CHART G. — The curve $A'B'C'D'E'$ is the normal curve of error of a coefficient of correlation for samples of 1,000 pairs of items.

puted from samples decreases rapidly as the size of the sample increases.

The coefficients of correlation for pig-iron production and interest rate on sixty-to-ninety day commercial paper for various degrees of lag, plus or minus the respective probable errors computed from the formula just given, are as follows:

$r_{0,0}$	$r_{0,3}$	$r_{0,4}$	$r_{0,5}$	$r_{0,6}$
$+.34 \pm .05$	$+.67 \pm .03$	$+.72 \pm .03$	$+.75 \pm .03$	$+.75 \pm .03$
$r_{0,7}$	$r_{0,8}$	$r_{0,9}$	$r_{0,12}$	
$+.73 \pm .03$	$+.70 \pm .03$	$+.65 \pm .03$	$+.45 \pm .05$	

The probable errors, considered in connection with the coefficients of correlation, enable us to judge the significance of the coefficients. Let us recall the general rule already cited that "when r is greater than, say, six times its probable error, we may be practically certain that the phenomena are not independent of each other, for the chance that the observed results would be obtained from unconnected causes is practically zero."¹ Applying this rule to the present case, we are safe in concluding, first, that there is a significant degree of positive correlation for pig-iron production and interest rate on sixty-to-ninety day commercial paper, and second, that interest rate lags behind pig-iron production five or six months.

Comparison of the cycles of pig-iron production and interest rate and also of various other series has indicated that it would be unprofitable to measure the lag of one series as compared with others in finer time units

than two months. Consequently, coefficients of correlation have been computed from averages of items for two-month periods, January and February, March and April, May and June, July and August, September and October, November and December.

The method followed, then, of ascertaining the degree and nature of the correlation between various pairs of series is as follows:

- (a) The graphs of *cycles* of business statistics were charted on tracing paper and then compared by placing one over the other on the glass top of an illuminated box. This preliminary comparison was made by three independent observers. Each observer recorded, first, the degree (high, moderate, low) and sign (+ or -) of the correlation for the closest fit of the two graphs; second, the amount of lag in months for the maximum correlation; and third, the consistency or systematic character of the lag. The results thus secured were used as a guide in further studies of correlation.
- (b) Coefficients of correlation were computed between those pairings of items which, from the preliminary comparison, appeared likely to result in maximum correlation. In order to avoid unnecessary computation the monthly items were combined to make bimonthly items. The lag, as shown by the highest coefficient of correlation, was thus measured to the nearest two months. The degree of correlation was not considered noteworthy unless r was upwards of $+.50$.

Having adopted a method of measuring correlation between the cycles of any two series, the next steps are to classify all the series with respect to correlation and lag, to decide what series constitute good indices of business conditions, and to construct from them one or more synthetic indices.

3. THE GROUPS OF SERIES: CONSTRUCTION OF INDICES OF GENERAL BUSINESS CONDITIONS

Comparison of the cycle chart of each of twenty series with the cycle charts of the remaining nineteen, as described in the preceding section, led to certain important conclusions. These conclusions are as follows:

The judgments of the three independent observers concerning the sign and degree of correlation, and approximate lag for best "fit" of the curves were, in the main, consistent. There were differences of opinion in several cases, however, as to the exact lag for maximum correlation. In a few cases there was uncertainty as to whether the best fit of the two cycle charts was positive for a lag in one direction or negative for a lag in the other direction. Furthermore, there was not always unanimity of opinion concerning the degree of correlation even in the rough classification of "high,

¹ Bowley, *Elements of Statistics*, 2d ed., p. 320.

moderate, or low." Sometimes, for instance, one observer would decide that the correlation between two graphs was "high" and the other observers would call it "moderate" or "moderate to high." In any case, the three-term classification (high, moderate, low) could only indicate the degree of correlation very roughly. Consequently, it was impossible, on the basis of the graphic comparison alone, to arrange or "array" the pairs of series compared in order of the magnitude of the correlation between the various pairs. The graphic comparison could be utilized, therefore, merely as a guide for further investigation of the questions at issue. In all cases it was clear that correlation coefficients must be computed before a final decision could be reached. In other words, decisions concerning the nature and degree of correlation between pairs of series were actually based upon graphic comparison as a first approximation and the coefficients of correlation as a second approximation.¹

Comparison of the series over the illuminated box was an essential part of the procedure. It showed that the cycles of the several series are related but the wave movements are neither synchronous nor always positively correlated; that there are various degrees of correlation and lag; that the waves in the charts do not occur simultaneously but consecutively; that for some pairs the lag is approximately constant, for other pairs variable. The graphic comparison supplemented by coefficients of correlation makes it possible to arrange the series in order corresponding to the time of occurrence of the crests and troughs of the waves.

The systematic movements of the series, with reference to each other, the nature of the correlation between pairs of series, and the length and direction of the lag were definitely interrupted with the outbreak of the war in August 1914. It may be said that, in general, the order of the series as to lag was completely reversed. That is to say, series such as commodity prices and interest rate, which usually rise and fall four to eight months after bank clearings and pig-iron production, were the first series to feel the effects of the economic cataclysm caused by the declaration of war. The value of building permits, which usually forecasts fluctuations in commodity prices and pig-iron production, lagged behind these series in the upward movement of 1915-16. Another interference with the orderly movement of the series came in the spring of 1917 when the United States entered the war. After that time the value of building permits ceased to be an index of current or prospective industrial activity. As we have seen, the slackening of production in the winter of 1917-18, resulting from transportation congestion and the fuel situation, is revealed by the marked temporary fall in bank clearings and pig-iron production. In short, the disturbance due to the war completely upset the orderly routine of

peace times.² In consequence of these considerations, comparison of the various series with each other for correlation and consistency of fluctuations is limited in the present issue of the REVIEW to the period ending July 1914.

The conclusions, just stated, bear directly on the question which we now propose to discuss. Is it legitimate to combine several series of business statistics into a synthetic index of business conditions, the fluctuations of which will measure, in normal peace times, more accurately than a single series, the intensity and ramification of industrial activity? In short, is it legitimate, for our purpose, to average several series?

It has been customary for commercial statistical agencies to average a number of statistical series to secure a "business barometer" or a "compositplot." For instance, one agency uses the following combinations:

1. A graph to show the price trend of thirty-two leading stocks.
2. A business graph based upon the statistics of bank clearings, railroad earnings, pig-iron production and prices, commodity prices, imports, building, and immigration.
3. A banking graph based upon reserves, deposits, the rate of commercial paper, the percentage of loans to deposits, and the percentage of reserves to loans.

Another agency combines the following groups of series and presents a *single* graph:

1. A group indicating mercantile conditions, based on statistics of immigration, new building, commercial failures, and bank clearings.
2. A group indicating monetary conditions, based on commodity prices, foreign trade, foreign money rates, and domestic money rates.
3. A group indicating investment conditions, based on conditions of leading crops, railroad earnings, political factors (estimated), and prices of stocks.

In neither case is any justification offered for the choice of series to be combined; in neither case is sufficient information given to enable one desiring so to do, to check up the computation. All that can be said is that available series are averaged by some process or other.

Inspection of the lists of series combined by the commercial agencies to secure "business barometers" reveals the fact that a most miscellaneous selection of series is averaged. No criterion of homogeneity of series is mentioned; apparently none is even conceived to be necessary. Our preliminary survey indicates that the series entering into the business barometer have dissimilarities no less marked than their similarities. Even though all possess wave movements, such movements differ in amplitude, intensity, and time of ebb and flow. To combine such series, it would appear, is to blur the picture and conceal its meaning, not to make its outline distinct and its message unmistakable.

¹ A table giving the reconciled judgments of three observers is given on pp. 184-187 following. All the coefficients of correlation computed are given on pp. 182, 183.

² The reader is referred to Part I, Section 4, for a discussion of the statistical effects of the transition from peace to war and war to peace.

Clearly, the first problem encountered in constructing a synthetic index of business conditions is to sort the various series according to (a) correlation, and (b) lag. In the preceding section we have described the method adopted of comparing series. Comparison of the cycles of the various series according to that method led to the conclusion that wave movements occur in the following order:

- I. 12. Rate (yield) on ten railroad bonds.
20. Price of twenty railroad stocks.
18. Price of industrial stocks.
- II. 6. Building permits.
1. New York clearings (standard for group).
8. Shares traded.
- III. 2. Production of pig iron (standard for group).
3. Outside clearings.
5. Imports.
9. Unfilled orders U. S. S. C.
11. Business failures.
- IV. 4. Bradstreet's prices (standard for all series).
16. Bureau of Labor prices.
7. Railroad gross earnings.
23. Reserves of New York banks.
- V. 17. Dividend payments.
22. Loans of New York banks.
24. Deposits of New York banks.
13. Rate on four-to-six-months paper.
14. Rate on sixty-to-ninety day paper.

It is probable that three questions come to the mind of the reader as he examines the list of series just given. The questions are: What is the reason for grouping the series in this peculiar fashion? What is the criterion used for the arrangement of the individual series and groups of series, first to last? What is the meaning of the phrases "standard for group" and "standard for all series"? These questions will now be considered.

The series of each group (I, II, III, IV, V) have cycles which move concurrently with the cycles of other series of the same group. That is, the maximum correlation for series of one group occurs when items for the same date are paired; moreover, the coefficient of correlation between any two series of a given group is significant. Thus, the coefficients of correlation among the series of Group II vary from $+.52$ to $+.84$; those of Group III vary from $-.57$ to $+.82$; and those of Group IV vary from $-.72$ to $+.77$. Table G (p. 183) gives the intra-group coefficients of correlation for concurrent items and for two months lag for each series as compared with the other, showing that the maxima occur for concurrent items.¹

What significance, if any, has the order of the groups in our list? The series of Group II lag behind the series of Group I by two to four months; those of Group III lag behind those of Group II by two to four months; those of Group IV lag behind those of Group III by two to four months; while those of Group V lag behind those of Group IV by four to six months. Table H

(p. 183) gives the inter-group coefficients of correlation for various pairings of selected series.

The series labeled "standard for group" were used as representative series of each group in the computation of intra-group and inter-group coefficients of correlation. Bradstreet's prices were selected as the standard with which to correlate all series. This selection of standard series was made, mainly on the evidence offered by the cycle charts, in order to decrease the number of coefficients of correlation computed; it was thought unnecessary to compute the coefficients for every possible pair of the twenty series examined.² It is obviously desirable to select a standard with which to correlate various series. Three such standards were chosen — bank clearings of New York City, tonnage of pig iron produced, and Bradstreet's index of commodity prices. One reason for this selection is that the cycles of these three series do not occur simultaneously. New York bank clearings were selected because the items fluctuate directly with the amount of credit instruments originating with loans and speculations as well as the physical volume and prices of commodities bought and sold. Tonnage of pig iron was selected because it appears to be the best single index of physical productivity. Bradstreet's index of commodity prices was selected because it is a composite of ninety-six wholesale prices of important raw materials and manufactured goods.³

The consensus of opinion of writers on business cycles is that such cycles are preëminently characterized by price movements, culminating in crises and reaching bottom in times of depression.⁴ Consequently, coefficients of correlation were computed for each of three pairings of items with Bradstreet's prices. The pairings made were as follows:

1. Pairs which appeared to have the maximum correlation.
2. Pairs obtained by shifting cycles two months to right.
3. Pairs obtained by shifting cycles two months to left.

The coefficients of correlation for (a) Bradstreet's prices as the standard and (b) nineteen other series as correlatives are given in Table F (p. 182). These coefficients, together with those of Tables G and H, give the requisite information for arraying the various series in order of the time of their wave movements and for grouping the series. The series of a given group may, legitimately, be averaged. We have computed five

¹ The number of groups is, of course, somewhat arbitrary. Likewise, there are "border-line" series which might be assigned to either of two adjacent groups.

² There are 190 possible pairs of series among the twenty. If three coefficients are computed for each pair 570 coefficients would be found. Since there are 138 items in each series a total of nearly 80,000 products would be required, a considerable task to compute, record, and add.

³ See paragraph on source and nature of data on p. 139 for a detailed description of this series. It is the plan of the REVIEW to make an intensive and comparative analysis of this and other price indices in a later number.

⁴ See *Memoranda*, p. 206 for the rôle of prices in the business cycle.

synthetic indices of business conditions by averaging the concurrent cycles of the series composing each of the five groups. These synthetic indices, presented graphically, picture the ebb and flow of various phenomena connected with the business cycle. The crests or troughs of the waves do not occur simultaneously, but successively over a period of twelve to sixteen months.

The synthetic indices interpreted with reference to each other and to the series composing each group constitute an index of general business conditions.

In the preceding sections of Part II we have explained the technique adopted for correcting individual series of business statistics and for measuring the correlation between series. As a measure of correlation we have used the coefficient of the English school of biometricians. Questions arise in connection with our use of the coefficient to measure correlation between time series of economic statistics. The coefficient of correlation was devised for another purpose. Is the use of it that has here been made a legitimate one? Furthermore, what, exactly, is the concept of correlation? What is the relation between correlation and causation? What is the nature of "economic law?" These questions, general in their nature, will be considered in the following section. That section may be safely omitted by readers who are interested in the results of statistical processes rather than in an extended consideration of the concepts at the basis of our statistical technique.

4. GENERAL CONSIDERATIONS RELATING TO THE MEASUREMENT OF CORRELATION

Perhaps an accurate concept of correlation and the coefficient of correlation may best be secured by examining briefly the development of the theory. The theory of correlation was originally developed by English scientists in connection with their studies of biological evolution. Sir Francis Galton deserves the credit for conceiving of a numerical measure of the intensity of correlation and for devising a practical statistical

method to secure such a measure.¹ He has concisely stated his point of view in the sentence which has been adopted as the motto of the Francis Galton Eugenics Laboratory: "Until the phenomena of any branch of knowledge have been submitted to measurement and number it cannot assume the status and dignity of a Science."

Sir Francis Galton, Karl Pearson, G. Udney Yule, and F. Y. Edgeworth deserve credit for developing the mathematical theory of correlation and for applying that theory. It should be said, however, that "the method of correlation is only an application to the purposes of statistical investigation of the well-known *method of least squares*. It is impossible, therefore, entirely to separate the special literature of the theory of correlation from that of the theory of error or of the method of least squares. Of the numerous memoirs on the theory of error the most important in the present connection is that of A. Bravais,² who as long ago as 1846 discussed the theory of error for points in space, regarding the errors as either independent or correlated, from the standpoint of the normal law of errors. He did not, however, use a single symbol for a correlation coefficient, although the product-sum formula may be regarded as due to him, in a sense — *i. e.*, the product sum is introduced into his expression for the frequency of various combinations of error."³ Bravais, obviously, was interested in the theory of correlation as a branch of pure mathematics, while Galton and the English school were primarily interested in the application of the theory.⁴

It was Karl Pearson and those working with him in the Biometric Laboratory who are chiefly responsible for insisting that, in our studies of inheritance, "we must proceed by the method of statistics, rather than by the consideration of typical cases."⁵

In view of the fact that the theory of correlation was developed as an aid in the study of evolution it is natural that correlation should first have been defined in biological terms. Thus Pearson says: "Two organs in the

¹ See Yule, "The Applications of the Method of Correlation to Social and Economic Statistics," in the *Journal of the Royal Statistical Society*, December, 1909, pp. 721-729, for a helpful summary of the history of the subject. This paper contains a selected list of thirty-nine memoirs on correlation arranged in chronological order under years of publication. The publications in which Sir Francis Galton first considered the problem of correlation with reference to the statistics of heredity are: "Family Likeness in Stature," *Proceedings of the Royal Society*, London, 40, p. 42 (1886), and appendix to the preceding paper by J. D. H. Dickson, *ibid.*, p. 63; "Correlations and Their Measurement, etc.," *Proceedings of the Royal Society*, London, 45, p. 135 (1888); and *Natural Inheritance*, (1889).

² A. Bravais, "Analyse mathématique sur les probabilités des erreurs de situation d'un point," *Académie des Sciences. Mémoires présentés par divers savants*. II^e Série, T. 9, p. 255 (1846).

³ Yule, *Journal of the Royal Statistical Society*, December, 1909, p. 722.

⁴ In reviewing the second edition of Czuber's *Wahrscheinlichkeitsrechnung*, in the *Journal of the Royal Statistical Society*, May, 1911, pp. 643-647, Mr. J. M. Keynes says: "This must long remain the standard treatise on the topics with which it deals. There is no work in English which covers the same ground, and it greatly excels in grasp and thoroughness the French treatises which challenge comparison with it. . . . But of the modern theory of correlation and of the central position which this now holds in English statistical theory,

there is no hint. . . . Professor Czuber's methods are in direct line of descent from those of the classical writers on Probability and Error, and they possess the style and lucidity which such a history naturally gives them. But the reader must feel that these methods have reached their limit of accomplishment, and that nothing very novel can result from attempts to perfect them further. Recent English contributions, on the other hand, fragmentary and often obscure or inaccurate though they now are, seem to have within them the seeds of further development and to carry the methods of mathematical statistics into new fields." The French have done very little in the field of correlation. For instance, the second edition of Poincaré's *Calcul des Probabilités* (1912) is characterized by Keynes as follows: "This book has no reference to any of the researches, either German or English, which seek by the union of Probability and Statistics to forge a new weapon of scientific investigation." (*Journal of the Royal Statistical Society*, December, 1912, p. 114.) The Italians and Russians have made some contributions to or applications of the theories of correlation developed by the English school.

⁵ Pearson, "Mathematical Contributions to the Theory of Evolution, III. Regression, Heredity, and Panmixia," *Philosophical Transactions of the Royal Society of London*, A. 187, p. 255 (1896). See numerous articles in *Biometrika* for illustrations of the methods used. Recent writers have pointed out that the consideration of typical cases, as well as the method of statistics, gives valuable results. See Raymond Pearl, *Modes of Research in Genetics* (1915).

same individual, or in a connected pair of individuals, are said to be correlated, when a series of the first organ of a definite size being selected, the mean of the sizes of the corresponding second organs is found to be a function of the size of the selected first organ. If the mean is independent of this size, the organs are said to be non-correlated. Correlation is defined mathematically by any constant, or series of constants, which determine the above function.

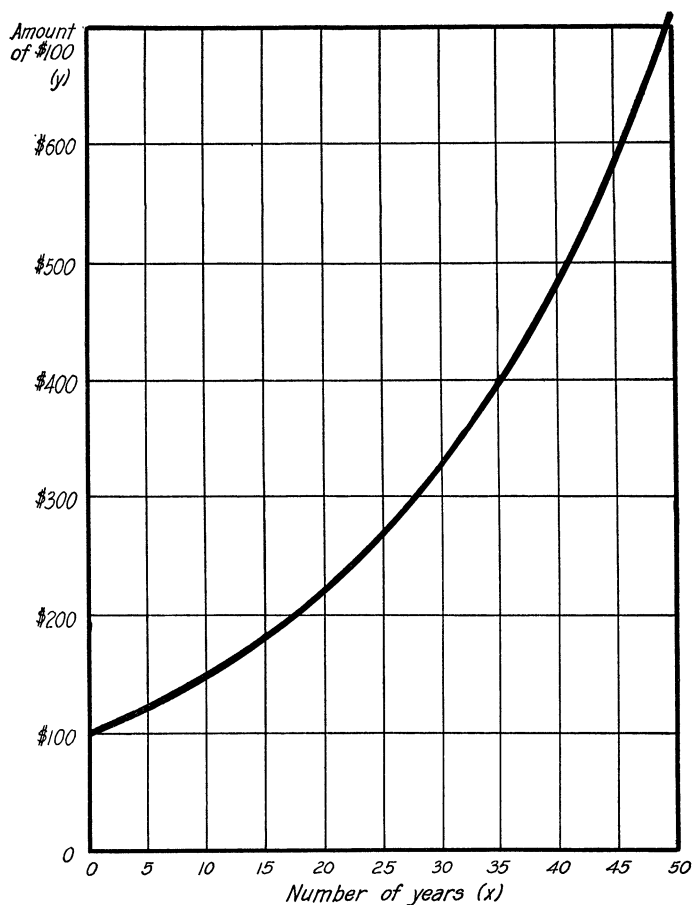
"The word 'organ' in the above definitions of variation and correlation must be understood to cover any measurable characteristic of an organism, and the word 'size' its quantitative value."¹ Pearson adds that "these organs may be in the same or in different individuals, or partly belong to one and partly to another individual. The complex may be constituted by a natural or artificial tie of any kind, but the tie is to remain the same for every complex, whether it be the result of mating or parentage, or from any physiological or social relation, etc."²

To illustrate the concepts just stated, suppose that we are attempting to answer the question, do tall fathers have tall sons? In this case, stature is the "measurable characteristic" in each of "a connected pair of individuals." Suppose the average stature of all adult males is sixty-six inches; suppose we select several thousand fathers whose stature is seventy-two inches or more, six inches above the average for all, and find the mean stature of the sons of this group of tall fathers to be sixty-nine inches, three inches above the average stature of all adult males. If similar results appear consistently for selected fathers and their sons, we may conclude that the stature of sons depends upon the stature of fathers; or, in other words, the stature of sons is a function of the statures of fathers; or, in still other words, the statures of fathers and sons are correlated. We may be able to state a "law" of the inheritance of stature, or give the stature of sons as a function of the stature of fathers. It is clear, however, that although tall fathers may, *in general*, have tall sons, an individual tall father may have a short son, or perhaps several sons, some tall, some short. That is, two concepts are involved; first, the law or function or equation expressing the relation *on the average*, existing between the two variables involved, and second, the degree with which individual cases adhere to the law.

To illustrate the first concept, it may be possible to say that for an average deviation in stature of fathers of n inches from the mean for adult males, the stature of the sons of those fathers will deviate in the same direction by $\frac{n}{2}$ inches. This is a law or function, the first concept that we have named.³ But the statement of the function does not describe the situation completely. How *accurately* does the function describe the situation; how *systematic* is the relationship between statures of fathers and sons; are the exceptional cases few or many? These are different forms of a question which requires a quantitative answer. Such an answer

is given by the coefficient of correlation. The coefficient is unity if there is no exception to the law of statures; it is zero if the statures of father and son are independent of each other; it is negative if tall fathers, in general, have short sons; it has a numerical value varying inversely with the degree of divergence (both in number of cases and magnitude) of the individual cases from a linear relationship.

It may be well to illustrate further the concepts just described. For instance, the compound interest law expresses the relationship between two variables, time and the amount of a given principal accumulated at



The amount (y) of \$100 at the end of each year (x), compounded at 4 per cent per annum.

a given rate of interest. The accompanying diagram shows the amount (y) of \$100 at the end of each year (x), compounded at 4 per cent per annum.⁴ Now if \$100 be kept continuously invested at 4 per cent per annum, and if there are no losses either of principal or interest, and if the accumulated interest is immediately reinvested at 4 per cent per annum, then, with these assumptions, the amounts shown by the curve will result at the times indicated. The assumptions named, however, are not apt to be fulfilled for long periods in the business world. The assumptions are artificial. Assume that

¹ Pearson, *ibid.*, p. 257. See also the historical summary on p. 261.

² *Ibid.*, pp. 261-262.

³ The law usually selected is the simplest, that is, the question usually put is this: On the average, does an increase or decrease of a given number of units in one series correspond to an increase or decrease (or inversely) of some constant number of units in the other series? This law is the one implied when the coefficient of correlation is used in the ordinary fashion. Artifices may be utilized for testing the adherence of measurements to more complicated laws. See Yule, *Theory of Statistics*, 1st ed., p. 202.

⁴ The algebraic expression of the law is $y = 100 (1.04)^x$.

thousands of investors having \$100 each, deposit their principal in various savings banks paying, at the start, 4 per cent; assume further that the accrued interest be deposited. There will be some total losses, some partial losses, and numerous changes of interest rates. Consequently, the actual accumulations of the depositors will be widely scattered. Possibly the points determined by the actual cases will not be grouped about the 4 per cent compound interest curve. Some other law may express more accurately the relationship between capital accumulated and number of years elapsed. The law, whatever it may be, is one concept, a statement of the accuracy or universality of the law is a correlative concept. The law may be an inaccurate one. Indeed, there may be such a low degree of correlation between the two variables, capital accumulated and number of years elapsed, that the statement of any law is futile.

Let us now return to the consideration of other concepts of the English school. In deriving the expression for the coefficient of correlation Pearson makes certain assumptions concerning the causes determining the sizes of the organs correlated. These assumptions are: first, "that the sizes of this complex of organs are determined by a great variety of *independent* contributory causes, for example, magnitudes of other organs not in the complex, variations in environment, climate, nourishment, physical training, and innumerable other causes, which cannot be individually observed or their effects measured;" second, "that the variations in intensity of the contributory causes are small as compared with their absolute intensity, and that these variations follow the normal law of distribution."¹ On the basis of these assumptions Pearson has devised the expression for r , the coefficient of correlation, which we have previously given (see p. 123).

It is clear that the statistical series of Galton and Pearson are unlike the time series of business statistics with which we are concerned at present. To be sure, the fundamental assumptions concerning the causes at work hold for economic as well as for biological data. The causes determining the monthly production of pig iron, for instance, or the interest rate, are very numerous. It is not easy, however, to establish the independence of these causes. Thus, the pig-iron production will be greater or less because of numerous causes, such as the labor conditions, the efficiency of foremen, variations of the railroad situation, the weather, the nature of the raw material, accidents of various kinds. With a *given* demand for pig iron at a *given* price there will be variation in production. But higher prices, or improved processes, may greatly stimulate production in old furnaces and cause the construction of new ones. A period of prosperity, perhaps, may be analogous either to a set of dependent contributory causes or to a cause with great variations in intensity.

Fortunately, in order to justify the use of the coefficient of correlation, it is not necessary to demonstrate that the highly refined assumptions used in the mathe-

matical analysis hold in the world of industry. The justification for the use of the coefficient of correlation in economics depends, rather, upon the empirical considerations given in the preceding section (Part II). There are certain important differences, however, between series of measurements of organs and indices of business activity that demand consideration.

The essential differences with reference to the problem of correlation between measurements of organs and the items of time series are the following:

1. The items of time series must be defined for a selected time unit; they measure one form of activity of industrial society, whereas biological measurements are individual observations of objects whose measurements are independent of time.

2. Because they are defined for units of time, the items of time series are *ordered* in time; each item has a definite position with respect to the other items. The way to compare the items of two such series is to set up a relationship in time; that is, to pair concurrent items, or those definitely related in time. When we are correlating pig-iron production and interest rate, for instance, we wish to know not only the degree of correlation for *one* set of pairs, but the degree of correlation for *various* sets. There are various options possible in pairing items. On the other hand, when we are considering the correlation between measurements of organs, such as the stature and weight of the same individuals, or statures of fathers and sons, there is a *unique* pairing of items determined by the problem. The questions, does weight vary directly with stature, and do tall fathers have tall sons, obviously cannot be answered by pairing the stature of one individual with the weight of another or the stature of a father with the stature of his neighbor's son. The concept of lag is thus peculiar to time series.

3. Because they are ordered in time, the adjacent items in time series may be determined by overlapping or persistent causes; a succession of items of similar sizes is the rule rather than the exception. Adjacent measurements of organs, on the other hand, are not so connected. Expressed in other words, the items of time series, even though corrected for secular trend and seasonal variation, are apt to show a high degree of correlation when paired with succeeding items of the same series, whereas the items of biological series show only a nominal degree of correlation when so paired. Thus, the coefficient of correlation for the cycles of pig-iron production of one month and that of two months following is $+.84$.

In the preceding paragraph "overlapping causes" were mentioned. The manifestation of such causes, it was stated, is similarity between contiguous items of a series. Since we have eliminated secular trend and seasonal variation from each series to secure the "cycles," we have, presumably, eliminated the effect of two classes of overlapping causes before we correlate the cycles of any two series. The coefficient of correla-

¹ Pearson, *op. cit.*, p. 262.

tion for two series of economic statistics will frequently be larger than it otherwise would be, however, because of the influence of the same set of overlapping causes operating upon both series during the business cycle. It does not appear legitimate, therefore, to compare coefficients of correlation obtained from time series of economic data with those obtained from biological measurements. Each set of coefficients measures a type of correlation peculiar to the data utilized. A coefficient of correlation computed from economic data is not significant merely because of its absolute size; it is significant because of its size as compared with the size of other coefficients computed for the same two series but for other pairings of the items.

It is because of the numerous sets of influences operating upon the items of time series as time goes on that such items must be corrected if we would isolate the effects of designated influences. In the present study we have corrected the items for secular trend and seasonal variation, and have found the correlation between the cycles. The correlation coefficient, thus found, measures the degree of similarity between the wave movements of two series. Comparison of coefficients computed for various pairings of items enables us to determine the lag for maximum correlation. A method of measuring another type of correlation between two time series has been suggested by R. H. Hooker.¹ The method "consists simply in calculating the correlation coefficients of the differences between successive values of the two variables, instead of the differences from the arithmetic means. . . . Correlation of the deviations from an instantaneous average (or *trend*) may be adopted to test the similarity of more or less marked periodic influences. Correlation of the difference between successive values will probably prove most useful in cases where the similarity of the shorter rapid changes (with no apparent periodicity) are the subject of investigation, or where the normal level of one or both series of observations does not remain constant."²

Computation of the correlation coefficients of the differences between successive monthly or quarterly cycles would undoubtedly give us reliable measures where the lag is invariable. In case, however, the lag of the correlative series varies, say from three to six months, but nevertheless there is always a lag, the coefficient computed for first differences will not indicate the fact. In other words, a low coefficient may occur when the cyclical fluctuations are similar but are not invariably separated by the same time interval. This is precisely the situation before us in the present study. The method of correlation adopted, therefore, as most satisfactory is that of comparison of the cycle charts over the illuminated box, the conclusions being checked by the computation of coefficients of correlation between the cycles.³

¹ "On the Correlation of Successive Observations," in the *Journal of the Royal Statistical Society*, December, 1905, pp. 696-703.

² *Ibid.*, pp. 697, 703.

³ The variate difference correlation method does not yield reliable results for series such as we are dealing with in this paper. See "The

What is the significance of a conclusion (based upon inspection of the curves and consideration of the coefficients of correlation) that the graphs of the cycles of two series of business statistics are highly similar? If the cycles of one series systematically precede the cycles of another series does it mean that the movements of the first *cause* the movements of the second? If so, how *much* similarity between the two series is necessary for us to term the first series "the cause" and the second series "the effect"? Or, are both series to be considered effects of a complex of joint causes? If so, are the dissimilarities due to differences in the constitution of the complex of joint causes determining each, to the varying intensity of like joint causes, or to both? Should we think of the various series as being *symptoms* of underlying economic conditions and of our problem as the sorting, analyzing, and judging of symptoms in order to secure an accurate description of those conditions? More specifically, what is the meaning of the similarity between the cycles of pig-iron production and the interest rate on sixty-to-ninety day commercial paper, the latter having a maximum correlation coefficient of +.75 for a six months lag?

In the present study we have made comparison of one series with another in order to secure a concise statement of *how* changes are taking place. The rationale we have used differs in no way from that of any other scientific investigation. The modern scientific point of view has been effectively stated by Karl Pearson. We cannot do better than quote his arguments.

"That a certain sequence has occurred and recurred in the past is a matter of experience to which we give expression in the concept *causation*; that it will continue to recur in the future is a matter of belief to which we give expression in the concept *probability*. Science in no case can demonstrate any inherent necessity in a sequence, nor prove with absolute certainty that it must be repeated. Science for the past is a description, for the future a belief."⁴ . . .

"In the struggle for existence man has won his dictatorship over other forms of life by his power of foreseeing the effects which flow from antecedent causes — not only by his memory of past experience, but by his power of codifying natural law, that is, by his power of generalising experience in scientific statements."⁵ . . .

"Anything is a cause which antedates or accompanies a phenomenon, and we ask if we vary that cause to what degree we vary or change the phenomenon. If we say the variation of the cause produces no effect on the phenomenon we have absolute independence; if we found variation of this cause absolutely and alone varied the phenomenon we should say that there was absolute dependence. Such absolute dependence of a

Variate Difference Correlation Method and Curve-Fitting" by Warren M. Persons, *Quarterly Publications of the American Statistical Association*, June, 1917.

⁴ Karl Pearson, *Grammar of Science*, 3d ed., Pt. I, p. 113 (1911).

⁵ *Ibid.*, p. 137.

phenomenon on a single measurable cause is certainly the exception, if it ever exists when the refinement of observation is intense enough. . . . But between these two limits of absolute independence and absolute dependence all grades of association may occur. When we vary the cause, the phenomenon changes, but not always to the same extent; it changes, but has variation in its change. The less the variation in that change the more nearly the cause defines the phenomena, the more closely we assert the association or the correlation to be. It is this conception of correlation between two occurrences embracing all relationship from absolute independence to complete dependence, which is the wider category by which we have to replace the old idea of causation. Everything in the universe occurs but once, there is no absolute sameness of repetition. Individual phenomena can only be classified, and our problem turns on how far a group or class of like, but not absolutely same, things which we term 'causes' will be accompanied or followed by another group or class of like, but not absolutely same things which we term 'effects'.¹ . . ."

"All classes of phenomena are linked together, and the problem in each case is how close is the degree of association. Likeness of causes produces likeness of effects; we can measure the degree of likeness, whether we are dealing with a chemical reaction or with the resemblance in any aptitude between parent and child. There is no question of absolute sameness in either case; there is a wide degree of difference in the likeness."² . . ."

"There is always in non-organic as in organic phenomena a residual variation. Repetitions are like within limits, but not same, for the antecedents are only like but never same. From this standpoint the universe appears as a universe of variation rather than as a universe controlled by the law of causation in its narrowest sense. No phenomena are causal; all phenomena are contingent, and the problem before us is to measure the degree of this contingency, which we have seen lies between the zero of independence and the unity of causation. That is briefly the wider outlook we must now take of the universe as we experience it."³

From this standpoint there is "no distinction in kind but only in degree between the data, method of treatment, or the resulting 'laws' of chemical, physical, biological, or sociological investigations. They all provide, or should provide, (i.) a conceptual routine, which is a functional expression of average experience, and

(ii.) a measure of the possible or probable deviations from this routine, which is a guide to the amount of variation in experience."⁴ . . ." This variation is small in physical experiences and large in economic experiences. In all types of experiences, however, our problem is to discover system in the midst of variation. Our problem in dealing with experience is not solved when we find a "functional expression of average experience"; we must also measure "the amount of variation in experience." Our problem does not even end here. The object of summarizing experience is to use that experience as a basis of future conduct or action. "Science cannot demonstrate that a cataclysm will not engulf the universe tomorrow, but it can prove that past experience, so far from providing a shred of evidence in favour of any such occurrence, does, even in the light of our ignorance of any necessity in the sequence of our perceptions, give an overwhelming probability against such a cataclysm."⁵ The probability that a discovered sequence will be repeated in the future depends, obviously, upon the breadth of the sample or the number of times the sequence has been observed and upon the variability of the sequence. Consequently, (i.) a "functional expression of average experience," (ii.) "the amount of variation in experience," and (iii.) "the probable error of experience"⁶ are all necessary in utilizing experience as a basis of action.⁷

In illustration of the points just made let us consider once again the relationship between the cycles of pig-iron production and the interest rate on sixty-to-ninety day commercial paper. Using the data for the period January 1903-July 1914, we find that pig-iron production varies directly with the interest rate six months later. The *linear* functional expression for interest rate (y) in terms of pig-iron production (x) is

$$y = +.758x + .061.$$

The coefficient of correlation of $+0.75$, being less than unity, expresses the fact that there is "variation in experience." The probable error of ± 0.03 is a measure of the breadth of the sample utilized for the coefficient just given. But are there not also "probable errors," so to speak, in the lag and in the type of mathematical function used? There are. The lag or interval varies from three to nine months; an interval of six months merely seems to be more persistent *during the period studied* than other intervals that may be selected.

the Misuse of Coefficients in this Connection" in the *Quarterly Publications of the American Statistical Association*, December, 1917, pp. 847-853. Dr. Willford I. King has concepts of economic law, causation, and correlation quite different from those defined by Karl Pearson and accepted by the present writer. Dr. King says, "A law which only works part of the time is no law at all. . . . There can be no such thing as imperfect or partial correlation. Every correlation to exist must be perfect. Every cause must produce its effect exactly and invariably" (pp. 848, 850). Dr. E. E. Day has taken issue with Dr. King (see *Quarterly Publications of the American Statistical Association*, June, 1918, pp. 115-118). The present writer agrees with Dr. Day that Dr. King's "discussion of the nature of correlation is so extreme and inflexible as to be wholly misleading" and that it "will add materially to the confusion regarding the nature of correlation and the correlation coefficient." *Ibid.*, pp. 115, 188.

¹ Karl Pearson, *Grammar of Science*, 3d ed., Pt. I, p. 113 (1911), p. 157.

² *Ibid.*, p. 166. ⁴ *Ibid.*, p. 173.

³ *Ibid.*, p. 174. ⁵ *Ibid.*, p. 113.

⁶ "The probable error is merely a quantity such that we may expect greater and less errors of simple sampling with about equal frequency, provided always that the distribution of errors is normal." Yule, *Theory of Statistics*, 1st ed., p. 307.

⁷ Some writers on economic statistics have confused the notions of correlation (closeness with which the actual corresponding fluctuations obey any law which we may select) and of the law which we select for the test. For instance, see the Note by James D. Magee and Rejoinder by Warren M. Persons in the *Quarterly Publications of the American Statistical Association*, December, 1914, pp. 345-350. In an article on "The Correlation of Historical Economic Variables and

TABLE K.—CORRELATION TABLE OF CYCLES FOR (a) MONTHLY PRODUCTION OF PIG IRON, JANUARY 1903–JANUARY 1914 AND (b) MONTHLY INTEREST RATE ON SIXTY-TO-NINETY DAY PAPER SIX MONTHS LATER, JULY 1903–JULY 1914

Production of Pig Iron																					(x)	
Corresponding Mean of (x) ↓	-3.0	-2.7	-2.4	-2.1	-1.8	-1.5	-1.2	-.9	-.6	-.3	.0	+.3	+.6	+.9	+1.2	+1.5	+1.8	+2.1	+2.4	+2.7	+3.0	← Measure (y)
	0	0	3	5	2	4	3	11	15	9	8	13	22	23	12	3	0	0	0	0	0	Totals ↓
																						0 + 3.0
+1.20															1							1 + 2.7
+ .90														1								1 + 2.4
+1.05														1	1							2 + 2.1
+ .90												1		11		1						4 + 1.8
+ .66													1111	1								5 + 1.5
+ .79											1	11	1	1111	11	1						11 + 1.2
+ .86											1		1111	111111	111	1						16 + .9
+ .75											1	11	1	111	111							10 + .6
+ .30							1		1			111	11111		1							11 + .3
+ .02								1	1111	1			111	111								13 .0
.00								11	1	1	11	111	1		1							11 -.3
-.40					1			111	111	1111	1	1	11									15 -.6
-.76			11			11	11	111	1111	1	1	1	1	1								17 -.9
-1.40			1	11	1	11		1		1	1											9 -1.2
-1.29				111				11	1	1												7 -1.5
																						0 -1.8
																						0 -2.1
																						0 -2.4
																						0 -2.7
																						0 -3.0
(x) ↑ Class Means (y) →			1.00	1.38	.90	1.05	.50	.76	.46	.70	.08	.30	.48	.94	.98	.130						Corresponding Mean of (y) ←

If the extent and nature of our data warranted the expenditure of further labor we might secure a more comprehensive description of the lag than one which merely is "six months on the average." In the present case, however, a more comprehensive description of the lag is not useful. Likewise, a more complicated function might be found which would describe the relationship for the period studied more accurately than one of the first degree. But we are looking for simple relationships. The more complicated the relationships the less valuable the results. It may be said that for over 95 per cent of economic series it is not worth while to search for a more complicated functional expression between the variables than one of the first degree. In order to make it entirely clear, however, that it is possible to use other than linear functions, a function of the second degree (a parabola) will be fitted to the paired items of pig-iron production and interest rate on sixty-to-ninety day paper.

Our present problem, then, is to fit a parabola (as well as the straight line already found) to the data for the production of pig iron and interest rate on sixty-to-ninety day commercial paper as given in the correlation table (Table K). Chart H (p. 136) presents the results. In fitting the parabola to the data, pig-iron production (x) was considered the independent variable, and interest rate (y) the dependent variable.¹ In other words, if we assign a given value to pig-iron production and substitute that value for x in the equation of the parabola, $y = -.074 + .829x + .147x^2$, the resulting value of y is our estimate of what the interest rate will be six months later. It must be remembered, however, that the correlation between pig-iron production and interest rate is

¹ The method of moments was used. The values of the constants in the equation $y = a + bx + cx^2$ were found by solving the following simultaneous equations:

$$\begin{aligned}\Sigma y &= na + b\Sigma x + c\Sigma x^2 \\ \Sigma xy &= a\Sigma x + b\Sigma x^2 + c\Sigma x^3 \\ \Sigma x^2y &= a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4.\end{aligned}$$

not perfect, and hence that the same interest rate does not invariably follow a *given* pig-iron production. There is more or less scatter or variability. The value of y that we get for a given x is really an *average rate of*

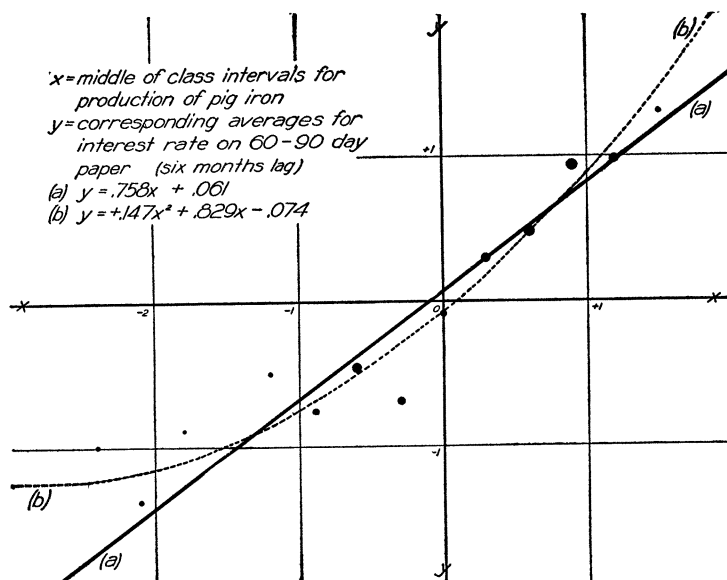


CHART H. — Comparison of the results of fitting the two functions for (a) a straight line, and (b) a parabola, respectively, to the data presented in Table K.

interest that corresponds to a *given* pig-iron production six months previous. In Chart H the solid circles are located by using as abscissas the mid-points ($-2.4, -2.1, \dots, 0, \dots, +1.2, +1.5$) of the class intervals of pig-iron production and as ordinates the corresponding *averages* of interest rate ($-1.00, -1.38, -.90, -1.05, -.50, -.76, -.46, -.70, -.08, +.30, +.48, +.94, +.98, +1.30$). The areas of the solid circles are proportionate to the frequencies (3, 5, 2, 4, 3, 11, 15, 9,

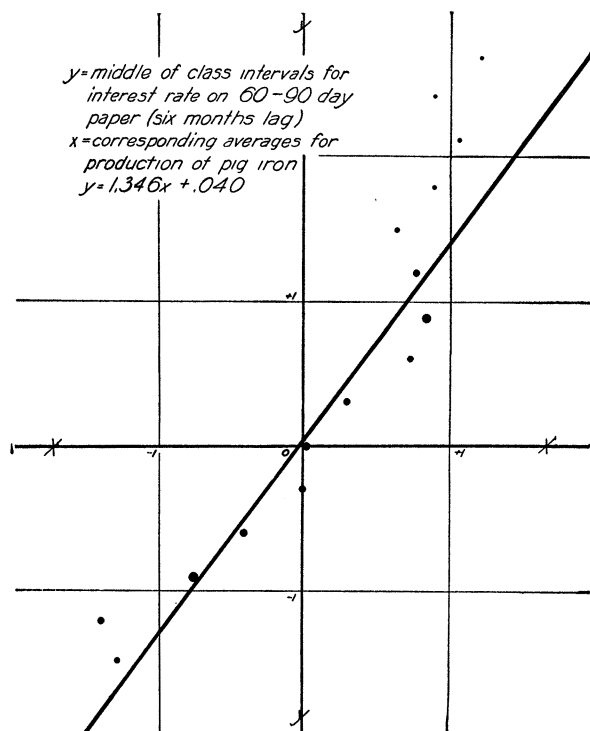


CHART J. — Straight line fitted to interest rate as the independent variable and pig-iron production as the dependent variable (Data of Table K).

8, 13, 22, 23, 12, 3). If all of these circles were located *exactly* upon the straight line or upon the parabola, our estimate of an *average* y for a *given* x , by means of the proper equation, would be exact. The precision of our

estimate varies directly with the closeness with which the circles are grouped about the line or curve. A decision as to the best function to use in any given case can usually be reached by simple inspection of the fit of the graphs to the points.

Thus far in our discussion of fitting a straight line and a parabola to interest rate and pig-iron production, the former has been taken as the dependent variable, and the latter as the independent variable; so that by use of the equation secured, the former could be estimated from the latter. Cannot we reverse this process and take interest rate as the independent variable? That is, cannot we estimate average production of pig iron from a given interest rate? We can, but not by use of the same function. Chart J (above) shows the straight line which we obtain by taking interest rate as the independent variable and pig-iron production

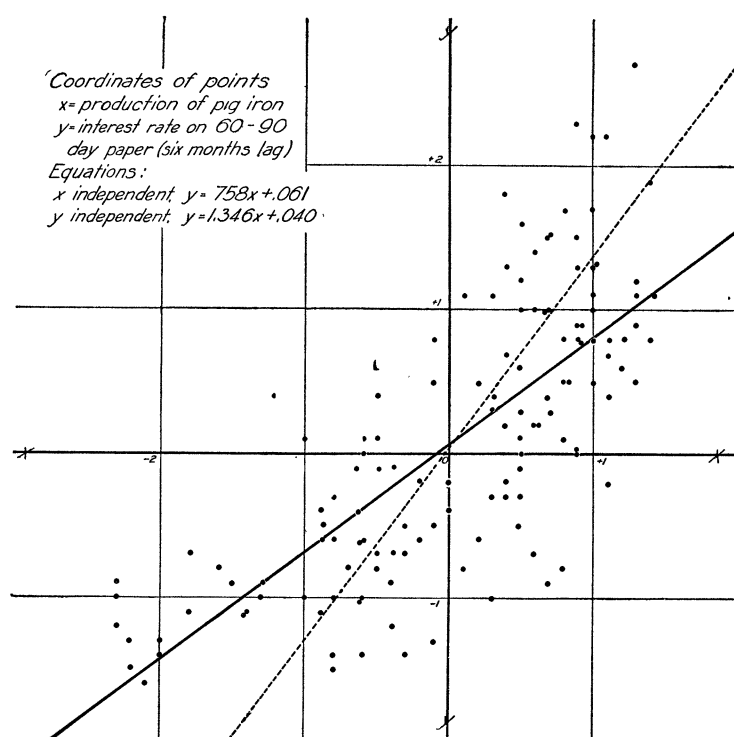


CHART K. — Points located by pairing items of (a) production of pig iron and (b) interest rate on sixty-to-ninety day paper six months later, together with the two lines fitted to the data of Table D.

as the dependent variable. In other words, the line is fitted to the solid circles whose ordinates are the mid-points ($-1.5, -1.2, \dots, 0, \dots, +2.4, +2.7$) of the class intervals of interest rate, whose abscissas are the corresponding averages of pig-iron production ($-1.29, -1.40, -.76, -.40, 0, +.02, +.30, +.75, +.86, +.79, +.66, +.90, +1.05, +.90, +1.20$) and whose areas are proportionate to the frequencies (7, 9, 17, 15, 11, 13, 11, 10, 16, 11, 5, 4, 2, 1, 1). The line in Chart J enables us to estimate *average pig-iron production* from a *given* *interest rate*. In like manner, instead of the straight line, a parabola or other curve might be fitted to these same points and be utilized for estimates.

The two lines which have been fitted (considering, first pig-iron production, and then interest rate six months later, as the independent variable) to the 133 points located by paired items are not identical. This lack of identity is due to the fact that the correlation between the series is not perfect. The more rigid the

correlation between the two series the closer will the two lines approach coincidence; when $r = \pm 1$ the lines will coincide. The 133 points, together with the two lines, are shown in Chart K.

To conclude this general discussion of correlation, it is necessary to consider one further point. In the derivation of the coefficient of correlation by Karl

diagram, however, is not marked enough to invalidate the present application of the Pearsonian coefficient of correlation to economic data.

Theoretically, the *irregular* fluctuations rather than the *cyclical* fluctuations are the ones which we should expect to be distributed according to the normal law of distribution of errors. In order to test this supposition, let us attempt to isolate the irregular fluctuations in a

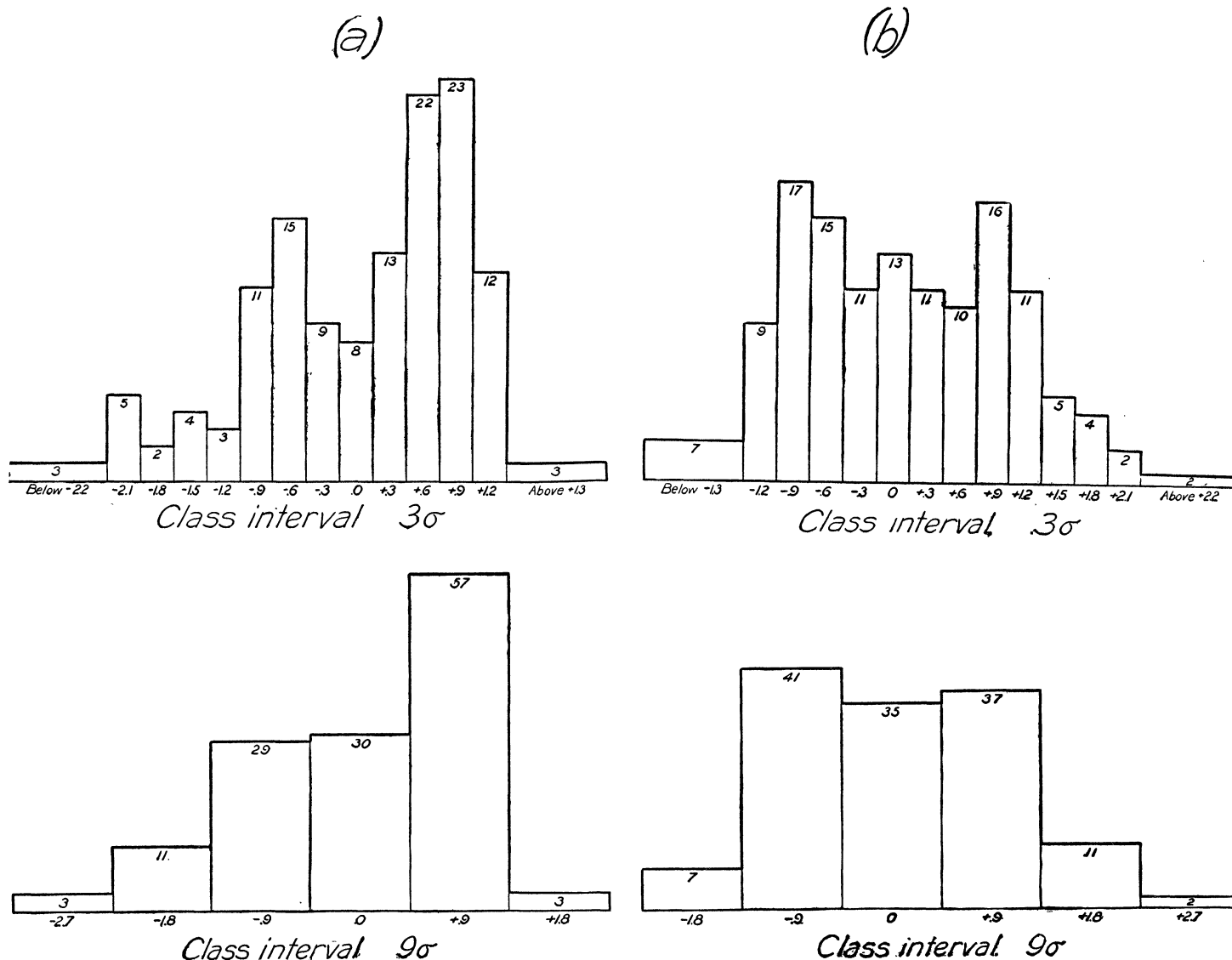


CHART L. — Frequency distributions of (a) pig-iron production, January 1903–January 1914, and (b) interest rate on sixty-to-ninety day paper, July 1903–July 1914 presented in the form of rectangular diagrams.

Pearson and others, it was assumed that the items of each series were distributed according to the normal law. Biological measurements, such as stature, weight, and intellectual capacity, are distributed according to that law. But are *economic* measurements, such as production of pig iron, interest rate on sixty-to-ninety day paper, and value of building permits, distributed according to the same normal law? Chart L (above) shows the frequency distributions of production of pig iron and interest rate on sixty-to-ninety day paper. For narrow class intervals, the rectangular diagram for the production of pig iron reveals a bimodal distribution; for wider class intervals the distribution is approximately normal. The same type of distribution, though less marked, is revealed by the diagram for interest rate on sixty-to-ninety day paper; in this case the distribution is “skewed” or unsymmetrical. This divergence from normal distribution shown in each

series wherein they appear to a marked degree, e. g., the value of building permits (Series 6).

The process adopted of isolating the irregular fluctuations for the value of building permits, 1903–16, is as follows:

(1) Monthly relatives were computed, each item being expressed as a relative with the corresponding ordinate of the secular trend as the base, thus presumably eliminating the secular trend.

Other relatives were then computed, by using the series described in the preceding paragraph as the numerators, and the appropriate indices of seasonal variation (the computation of which is described in Part III, Section 1 of this paper) as the denominators. This operation presumably eliminates, in addition to the secular trend, the seasonal variation, but it does not eliminate the cycles.

(2) There were computed the percentages of the

TABLE L. — IRREGULAR FLUCTUATIONS * OF MONTHLY VALUES OF BUILDING PERMITS ISSUED FOR TWENTY LEADING CITIES

(FREQUENCY TABLE, BY MONTHS, JULY 1903-JUNE 1916)

Irregular Fluctuations, Percentages	FREQUENCY												
	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Total
+46.0								1					1
+35.0												1	1
+32.3												1	1
+21.9		1			1		1					1	4
+17.9					1		1	2	1	1	2		8
+13.9	1	1	1	1		3			2	1			10
+ 9.9	1		2	1	1	3	2	1	1	1	1	1	15
+ 5.9	3	2		3	2	2	4	2	2	3			23
+ 1.9	3	2	1	1	1	3	3	2	2		4	1	23
- 2.1		2	2	3	2			3	2	3	2		19
- 6.1	1	1	2		2	1	1	2	1	3		5	19
-10.1	2		4	3	2		1			1	1	1	15
-14.1	1	2				1			2		1	2	9
-18.1		1	1	1							1		4
-22.1	1	1			1								3
-26.1											1		1
Total.....	13	13	13	13	13	13	13	13	13	13	13	13	156

* See text p. 137 for explanation of the method of securing irregular fluctuations. The seven irregular fluctuations greater than +17.9 are for the following dates: August 1911 (+46.0), December 1912 (+35.0), December 1913 (+32.3), December 1915 (+21.7), July 1914 (+20.3), February 1909 (+20.2), May 1907 (+18.2).

The eight fluctuations less than -18.0 are for the following dates: November 1914 (-27.1), May 1911 (-25.0), February 1904 (-23.7), January 1908 (-23.2), March 1908 (-21.8), February 1908 (-21.7), April 1916 (-19.6), November 1907 (-18.5).

twelve months moving averages to the corresponding ordinates of the secular trend as bases. This process (by averaging) presumably eliminates the irregular variations and seasonal variations; and (by taking ratios) the secular trend, but it does not eliminate the cycles.

(3) The items found in (2) above were subtracted from the items described in (1). The irregular variations were present in (1) but not in (2). The resulting differences are, presumably, approximations to the irregular fluctuations.

(4) The differences found above were tabulated in the form of a frequency table (Table L). From the

data in this table there was drawn a rectangular diagram. Finally, to this diagram there was fitted a normal curve (Chart M).¹

It is evident from Chart M that the distribution of the irregular fluctuations of the value of building permits is normal. The supposition with which we started is, therefore, confirmed.

In the preceding sections of Part II we have described the methods which we have adopted of measuring the correlation between statistical series. We have also analyzed the nature of the concepts and arguments involved in those methods. In Parts III and IV we shall

¹ Let $x_1, x_2, x_3, \dots, x_n$ be the actual or original series and $o_1, o_2, o_3, \dots, o_n$ be the corresponding ordinates of linear secular trend. Then, the secular trend is eliminated (by the process of dividing) in the series of ratios, $\frac{x_1}{o_1}, \frac{x_2}{o_2}, \frac{x_3}{o_3}, \dots, \frac{x_n}{o_n}$. Further, let $s_1, s_2, \dots, s_{12}$ be the monthly indices of seasonal variation. Then, both the secular trend and seasonal movement (but not the cyclical movement) are eliminated in the series of ratios

$$\frac{x_1}{o_1 s_1}, \frac{x_2}{o_2 s_2}, \dots, \frac{x_{12}}{o_{12} s_{12}}; \frac{x_{13}}{o_{13} s_1}, \frac{x_{14}}{o_{14} s_2}, \dots, \frac{x_{24}}{o_{24} s_{12}}; \text{ etc.} \quad (1)$$

Still further, let $m_7, m_8, \dots, m_{n-6}$ be the adjusted twelve months moving averages of the original items. Thus,

$$m_7 = \frac{1}{2} \left(\frac{x_1 + x_2 + \dots + x_{12}}{12} + \frac{x_2 + x_3 + \dots + x_{13}}{12} \right) \\ = \frac{x_1 + 2x_2 + 2x_3 + \dots + 2x_{11} + 2x_{12} + x_{13}}{24}, \text{ and} \\ m_{n-6} = \frac{x_{n-12} + 2x_{n-11} + 2x_{n-10} + \dots + 2x_{n-2} + 2x_{n-1} + x_n}{24}.$$

Any given moving average, m_i , corresponds to an original item, x_i , and to an ordinate of secular trend, o_i , with the same subscripts. In

the moving averages, small irregular variations and seasonal variations are "smoothed" out by the process of averaging; the secular trend and the cyclical movement (including the persistent irregular variations) are retained. If we take the ratios

$$\frac{m_7}{o_7 s_7}, \frac{m_8}{o_8 s_8}, \dots, \frac{m_{n-6}}{o_{n-6} s_{n-6}}, \quad (2)$$

the secular trend will be eliminated by the process, leaving the wave-like or cyclical movements isolated.

To summarize, in series (1) the secular trend and seasonal variation are eliminated, leaving the cyclical and irregular fluctuations as a residuum. In series (2) the secular trend, seasonal variation, and irregular variations are eliminated, or nearly eliminated, leaving the cyclical fluctuations isolated. Consequently, the differences between corresponding items of series (1) and series (2), which are

$$\left(\frac{x_7}{o_7 s_7} - \frac{m_7}{o_7} \right), \left(\frac{x_8}{o_8 s_8} - \frac{m_8}{o_8} \right), \dots, \left(\frac{x_{n-6}}{o_{n-6} s_{n-6}} - \frac{m_{n-6}}{o_{n-6}} \right)$$

constitute approximations to the irregular fluctuations. In other words, the irregular fluctuations are thus isolated. (We might, of

course, have taken the ratios $\frac{x_7}{m_7 s_7}, \frac{x_8}{m_8 s_8}, \dots$ instead of the differences and secured approximately the same results.)

give the results of applying the methods to numerous series of business statistics. Part III gives an outline of the method of correcting individual series for secular trend and seasonal variation, and the results of the

and seasonal variation, as follows: each monthly ordinate of secular trend is multiplied by the index of seasonal variation for that month; the resulting product is subtracted from the corresponding original item, and then expressed as a percentage of the ordinate of secular trend. We thus secure "percentage deviations of the original items from the corresponding ordinates of secular trend corrected for seasonal variation."

(e) The percentage deviations of the various series are expressed in terms of their respective standard deviations, in order to secure comparable cyclical fluctuations.

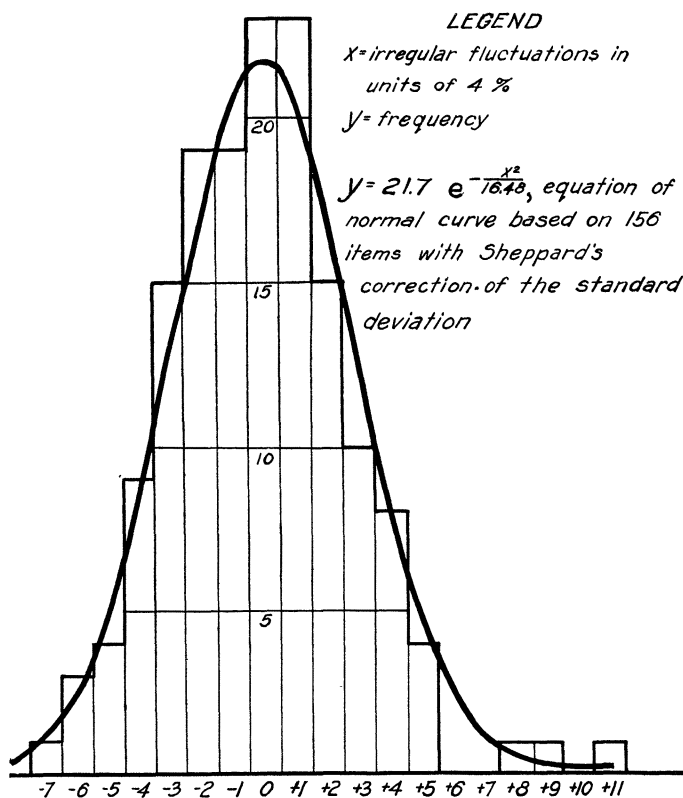


CHART M.—Rectangular diagram of the irregular fluctuations of monthly values of building permits issued for twenty leading cities (Table L) and the normal curve of distribution fitted to same, the irregular fluctuations being secured by correcting the original items for secular trend, cyclical fluctuations, and seasonal fluctuations.

application of that method.¹ Part IV gives an outline of the method of comparing the corrected series and the results of the application of that method. A general statement of the results secured in Parts III and IV is given in Part I.

III. APPLICATION OF THE METHOD TO THE DATA, (A) THE INDIVIDUAL SERIES

1. OUTLINE OF THE METHOD

(a) The data necessary for the application of the method are homogeneous monthly series of statistics covering a period of, say, fifteen years or more.

(b) The linear secular trend is found by fitting a straight line to annual items by the method of least squares. The compound interest curve, parabola, or other curves are used in case a straight line does not give satisfactory results.

(c) Indices of seasonal variation are found by taking the medians of month-to-month percentages for each of the twelve months. The medians, thus found, are progressively multiplied to form a continuous series, the discrepancy due to secular trend is distributed, and the average is made equal to 100.

(d) The original items are corrected for secular trend

¹ The results for Series 1 to 15 are given in the January REVIEW, pp. 37-48.

2. SUMMARY OF RESULTS FOR EACH SERIES

The source and nature of the series presented graphically in Charts 16 to 18, and 20 to 24 are given in the following summary. Additional information is given for Series 4 and 9. The numbers of the series (in parenthesis) are the same as those of the charts and frequency tables constructed from such series. Tables of the original series, the moving averages, the month-to-month link relatives, and the percentage deviations of the original items from the corresponding ordinates of secular trend corrected for seasonal variation are given in connection with Charts 16 to 18, and 20 to 24. The percentage deviations, just named, are presented graphically in Charts 116 to 118, 120, and 122 to 124, the units used being the respective standard deviations. The digits, in the units and tens places of the chart numbers, correspond to the number of the series upon which they are based. (For instance, Chart 16, Table 16, and Chart 116 all pertain to Series 16.) The equations of the lines of secular trend and the standard deviations for each series are given in the summary for that series. The indices of seasonal variation are given in the frequency tables of link relatives; graphs of the indices are presented as inserts of Charts 16 to 18, and 20 to 24.

4. BRADSTREET'S MONTHLY INDEX OF COMMODITY PRICES

SOURCE AND NATURE OF DATA. As stated in the REVIEW of January 1919 (p. 41), this series has been transcribed from summaries given in *Bradstreet's*. The following description is adapted from "Index Numbers of Wholesale Prices in the United States and Foreign Countries," in *Bulletin of the United States Bureau of Labor Statistics*, No. 173, pp. 109-112, 141-148.

The "index" is the aggregate of the per pound wholesale prices of selected staple commodities. The source of these quotations is not disclosed, but it is stated that they are from the primary markets. The index, expressed in dollars and cents, is published as of the first of each month in the department of *Bradstreet's* called "Measures of Movements." No revision of the method of constructing the index number appears to have been made since December 16, 1905. The exhibit as then published contained the index number by quarters from January 1, 1892 to October 1, 1898, and